On the interpretation of Langmuir probe data inside a spacecraft sheath

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(Received 16 April 2010; accepted 2 August 2010; published online 19 October 2010)

If a Langmuir probe is located inside the sheath of a negatively charged spacecraft, there is a risk that the probe characteristic is modified compared to that of a free probe in the ambient plasma. We have studied this probe-in-spacecraft-sheath problem in the parameter range of a small Langmuir probe (with radius \( r_{\text{LP}} \ll \lambda_D \)) using a modified version of the orbit motion limited (OML) probe theory. We find that the ambient electron contribution \( I_e(U_{\text{LP}}) \) to the probe characteristic is suitably analyzed in terms of three regions of applied probe potential \( U_{\text{LP}} \). In region I, where the probe is negatively charged (i.e., \( U_{\text{LP}} < U_1 \), where \( U_1 \) is the potential in the sheath at the probe position), the probe characteristic \( I_e(U_{\text{LP}}) \) is close to that of OML theory for a free probe in the ambient plasma. In the probe potential range \( U_{\text{LP}} > U_1 \), there is first a transition region II in applied potential, \( U_1 < U_{\text{LP}} < U_2 \), in which the key factor to determine the shape of \( I_e(U_{\text{LP}}) \) is a potential minimum \( U_M \) between the probe and the ambient plasma. This minimum gives the depth \( U_{\text{LP}} - U_M \) of a potential barrier that prevents the lowest energy ambient electrons from reaching the probe. For a high enough positive probe potential, in region III, the barrier becomes small. Here, \( I_e(U_{\text{LP}}) \) again approaches OML theory for a free probe. The boundary \( U_2 \) between regions II and III is somewhat arbitrary; we propose a condition on the barrier, \( U_{\text{LP}} - U_M \approx k_B T_e / e \), as the definition of region III. The main findings in this work are qualitative rather than quantitative. The existence of the transition region points to that special care must be taken to extract plasma parameters from measured \( I(U_{\text{LP}}) \) as the probe characteristic is likely to depart from usual OML in crucial respects: (1) the ambient plasma potential \( U_{\text{pl}} \) falls into the transition region, but there is no obvious knee or other feature to identify it, (2) there is in this region no exponential part of \( I_e(U_{\text{LP}}) \) that can be used to obtain \( T_e \), instead, (3) the probe size is important in determining the curve shape. We have tentatively applied our simplified probe-in-sheath model to real probe data from the Cassini spacecraft, taken in the dense plasma of Saturn’s magnetosphere. We propose that our model gives a better description than OML of measured Langmuir probe sweeps in space plasmas where the Langmuir probe is situated within the spacecraft sheath, i.e., for long Debye lengths. The understanding of these probe sweep effects in such regions may improve by self-consistent particle simulations of the spacecraft environment. © 2010 American Institute of Physics. [doi:10.1063/1.3482155]

I. INTRODUCTION

We consider the evaluation of ambient plasma parameters, such as potential \( U_{\text{pl}} \), electron density \( n_e \), and electron temperature \( T_e \), from spacecraft Langmuir probe data. With spinning spacecrafts, probes in the spin plane can be put outside the spacecraft sheath by using centrifugally stretched wire booms of sufficient length \( l_{\text{boom}} \gg \lambda_D \) or \( l_{\text{boom}} \gg r_{\text{SC}} \), where \( \lambda_D \) is the Debye length of the ambient plasma and \( r_{\text{SC}} \) is a characteristic radius of the spacecraft, so that either the probe is Debye screened from the spacecraft, or the spacecraft is small enough not to have a long-range influence anyway. If the boom is short, as may become the case when the probe is mounted on a three-axis stabilized platform, the probe may in certain cases be found within the spacecraft sheath. The local potential at the probe position, which we denote by \( U_1 \), will differ then from the ambient plasma potential \( U_{\text{pl}} \). We use a simplified geometrical model to illustrate how the potential structure in the spacecraft-and-probe system depends on the potentials of spacecraft and probe and on the length scales involved, and present an analytical model for a first-order estimate of the ambient electron collection \( I_e(U_{\text{LP}}) \) in the same geometry. We assume the parameter regime of collision-free plasmas and small probes (with radius \( r_{\text{LP}} \ll \lambda_D \) and use a modified version of the orbit motion limited (OML) probe theory for electron collection. The main findings are qualitative rather than quantitative. The existence of a transition region above \( U_1 \) points to that special care must be taken to extract plasma parameters from measured \( I(U_{\text{LP}}) \) as this curve likely departs from usual OML theory in important respects: (1) the ambient plasma potential \( U_{\text{pl}} \) has no obvious knee or other feature to identify it, (2) there is above \( U_1 \) no exponential part of \( I_e(U_{\text{LP}}) \) that can be used to obtain \( T_e \), instead, (3) the curve shape depends strongly on the probe size in a way that, for reliable quantitative evaluation, must be separated from the dependencies.

0034-6748/2010/81(10)/105106/8/$30.00 81, 105106-1 © 2010 American Institute of Physics
on $n_{e0}$ and $T_e$. The paper is organized as follows. In Sec. II we summarize for reference how plasma parameters $U_{pl}$, $n_e$, and $T_e$ are extracted from Langmuir probe characteristics $I_e(U_{LP})$ in standard OML theory, and in Sec. III we discuss the effect that arises when the probe is in the spacecraft sheath. Section IV describes a simplified model to deal with that problem, and in Sec. V a comparison is made between that model’s predictions and real space data. Section VI finally contains a summary and discussion.

II. DERIVATION OF PLASMA PARAMETERS FROM PROBE CHARACTERISTICS

Figure 1(a) shows the electron current to a spherical Langmuir probe, with a small radius $r_{LP}<\lambda_D$, in a homogeneous and collisionless plasma. In this parameter regime, the OML theory applies and the probe characteristic $I_e(U_{LP})$ is given by

$$I_e(U_{LP}) = \frac{\sqrt{8\pi r_{LP}^2 e^2}}{m_e} \sqrt{\frac{k_B T_e}{e}} e^{e(U_{LP} - U_{pl})/(k_B T_e)}$$

when $U_{LP} < U_{pl}$,

$$I_e(U_{LP}) = \frac{\sqrt{8\pi r_{LP}^2 e^2}}{m_e} \sqrt{\frac{k_B T_e}{e}} \left(1 + e^{(U_{LP} - U_{pl})/(k_B T_e)}\right)$$

when $U_{LP} > U_{pl}$. (1)

The OML theory needs modification when the area influenced by the probe, including the sheath around it, is not sufficiently small compared to the Debye length. In the present study we disregard such effects. The desired plasma parameters we will discuss in this work are the ambient plasma potential $U_{pl}$, the electron density $n_{e0}$, and the electron temperature $T_e$. These are obtained in OML theory as the combination of $(U_{pl}, n_{e0}, T_e)$ in Eq. (1) that gives the best fit, according to some suitable criterion, to a measured probe characteristic $I(U_{LP})$, with due consideration of contributions to the probe current other than ambient electrons. Although such fits might deal exclusively with the $I(U_{LP})$ curve, it is important to realize that information-carrying features of the probe characteristic are more clearly seen in the derivative $dI/dU_{LP}$ as illustrated in Fig. 1(b): (1) the slope of each individual part of the curve for $U_{LP}<U_{pl}$ can in principle give $T_e$; (2) the location of the knee in the derivative gives $U_{pl}$, and (3) the value of the derivative at the knee gives the combined quantity $n_{e0}/\sqrt{T_e}$. The coupling between these three features in the $dI/dU_{LP}$ curve and the plasma parameters is fundamental. We propose that, if they are uncertain or ambiguous in a measured probe characteristic, it is difficult to find an objective criterion to define the best fit of Eq. (1).

III. POTENTIAL STRUCTURE AROUND A BIASED LANGMUIR PROBE IN A SHEATH

To bring forth the main features, we use a simplified geometry: a spherical spacecraft of radius $r_{SC}$, with a spherical probe with radius $r_{LP}$ at a distance $\ell_{boom}$ from the spacecraft surface. We disregard (1) the influence on the potential by the probe boom, (2) wake effects, (3) the depletion of particles in the sheath due to absorption on the spacecraft, and the probe. The potential is under these approximations the same as that from two screened point charges. For example, the undisturbed potential $U_1$ at the probe position is found from the potential field $U(r)$ around single point charge $q$, representing the spacecraft,

$$U(r) - U_{pl} = \frac{q}{4\pi \epsilon_0 r} e^{-r/\lambda_D},$$

with the charge $q$ chosen such that at the spacecraft radius, $r=r_{SC}$, $U(r)$ coincides with the specified spacecraft potential $U_{SC}$.

$$U_{SC} - U_{pl} = U(r_{SC}) - U_{pl} = \frac{q}{4\pi \epsilon_0 r_{SC}} e^{-r_{SC}/\lambda_D},$$

so that $U(r)$ can be expressed in terms of $U_{SC}$ instead of $q$,

$$U(r) - U_{pl} = \frac{r_{SC}}{r} e^{-r_{SC}/\lambda_D} = \frac{r_{SC}}{r} e^{-(r-r_{SC})/\lambda_D}.$$ (4)

The sought potential $U_1$ is now simply $U(r)$, evaluated at the probe position $r=r_{SC}+\ell_{boom}$,

$$U_1 = U(r_{SC} + \ell_{boom}) = U_{pl} + (U_{SC} - U_{pl}) + \frac{r_{SC}}{r_{SC} + \ell_{boom}} e^{-\ell_{boom}/\lambda_D}.$$ (5)

Figure 2 shows the potential structure around a negatively charged spacecraft as we will consider here, and defines the variables to be used in this work. The key feature in Fig. 2 is a potential minimum $U_M$ (at the X-point of the equipotential curves) that arises for positively charged probes, i.e., when $U_{LP}>U_1$. The electron collection current $I_e(U_{LP})$ will depend on the depth and shape of this minimum, since it acts as a barrier and prevents all ambient electrons with energy be-
FIG. 2. (Color online) The potential structure in a simplified spacecraft-and-probe geometry and definitions of the parameters $U_{SC}$, $U_{LP}$, $U_{pl}$, $U_{1}$, $U_{SC}$, $r_{LP}$, and $\ell_{beam}$, to be used in this work. (a) Lines of constant potential around a negative spacecraft and a probe that is more positive than its surroundings. The probe in this figure thus attracts electrons. (b) Detail of the same potential structure as in (a), in the area around the probe. (c) The same potential as in (a), evaluated along the common axis of the spacecraft and the probe. The potential $U_{1}$ is the potential that would be found at the center of the probe position, if the probe was removed and only the spacecraft remained. A potential minimum, denoted $U_{SC}$, acts as a barrier that keeps the lowest energy ambient electrons from reaching the probe. This reduces the electron collection current below that of a free probe, held at the same potential in the ambient plasma.

low $e[U_{M}−U_{pl}]$ from reaching the probe. The complications are two: first, the depth and spatial width of the minimum depend both on the potential differences $U_{SC}−U_{pl}$, $U_{1}−U_{pl}$, $U_{LP}−U_{pl}$, and the involved scale lengths $\lambda_{D}$, $r_{SC}$, $\ell_{beam}$, and $r_{LP}$. Second, in this asymmetric potential structure, both the ambient electrons’ entry through the barrier and their collection at the probe are complicated problems, probably not tractable analytically, especially not for realistic spacecraft geometries.

Let us first look at the dependence of the potential minimum on the probe radius $r_{LP}$. It is illustrated in Fig. 3 which shows the radial potential profiles close to the probe for a reference case, with the parameters chosen to be relevant for the Cassini probe data to be presented in Sec. V. These parameters are referred to as the Cassini standard case in Table I. Apart from the actual Cassini probe size, $r_{LP}=25$ mm, Fig. 3 also shows the situation with two smaller probes. The key feature in these curves is the potential barrier $|U_{M}−U_{pl}|$ against ambient electrons. Although all three probes apparently are small compared to other involved dimensions, obeying $r_{LP} ≤ 25$ mm $≤ \min(\ell_{beam}, \lambda_{D})=1.3$ m, the barrier varies considerably with the probe size. For example, the uppermost curve in each panel of Fig. 3 shows the potential profiles for the same applied probe potential $U_{LP}−U_{pl}=30$ V. At this potential, the barrier is 1.35 V for the smallest probe, and rejects a large part of the ambient electrons, while the barrier for the real-sized 25 mm probe is only 0.35 V and lets most of them in. The origin of this strong dependence on probe size, even within the regime of $r_{LP} ≪ \lambda_{D}$, lies in that for a fixed probe potential, the probe carries a charge that is proportional to $r_{LP}^2$. So does, for example, the probe charge in Fig. 3 increase tenfold for each tenfold increase of $r_{LP}$. Due to its stronger charge, a larger probe thus has a greater influence over the potential field and over a larger region, pulling the minimum potential toward zero and pushing its location outward. Similar to the spacecraft, the probe contribution to the potential field is $(U_{LP}−U_{1})(U_{LP}/r)\exp(-(r−r_{LP})/\lambda_{D})$, where $r$ is the distance from the probe center. Here, the discussed size scaling is evident in the factor $r_{LP}/r$ and it is also clear that not even an infinite $\lambda_{D}$ will eliminate this effect.

We can, based on Figs. 3 and 4, make a first qualitative discussion of the probe-in-sheath effects on the information-carrying features in the derivative $dU/dU_{LP}$ of the characteristics. Consider first probes that are held at, and below, the local potential $U_{1}$. In this range of $U_{LP}$, there is no minimum, and the potential is monotonically decreasing from the ambient plasma to the probe. Since a Maxwell distribution retains its shape in a repulsive potential, although its density is

![FIG. 3. A demonstration of how the depth of the potential minimum $U_{M}$ depends on variations in the probe radius $r_{LP}$, with $x$ being the distance from the spacecraft center. The parameters $T_{e}$, $n_{e}$, $\lambda_{D}$, $r_{SC}$, and $\ell_{beam}$ are those of the Cassini standard case in Table I. The figures show potential profiles of the same kind as in Fig. 2(c) near the probe for varied applied probe potentials $U_{LP}$. In the case of a very small probe, with its small region of influence, the potential minimum changes only marginally with the probe bias. A smaller probe, therefore, to a larger extent experiences only the local potential $U_{1}$ and local, reduced density $n_{e}$. In the case of the largest probe (with $r_{LP}=25$ mm as in the Cassini standard case), the potential barrier almost disappears for the highest applied probe potential $U_{LP}=U_{SC}+30$ V, and the probe becomes much more exposed to the ambient plasma.](image-url)
scaled down, the probe should here experience electrons with the ambient temperature $T_e$ and density $n_{e1}$, yielding the OML current with this temperature and density. Note that since the relevant potential difference to use in the OML model here is $U_{LP} - U_1$, this current is the same as that of a free probe in the ambient plasma, i.e., given by Eq. (1) (we here neglect that a fraction of electrons, of the order of $\sqrt{m_e/m_0}$, have overcome the repulsive potential and hit the spacecraft and are therefore missing). Let us call this region I of the probe sweep. Next, let us consider the other extreme, probes held at high positive potential. For a probe with radius 25 mm, we saw in Fig. 3 that the barrier becomes only 0.35 V at 30 V positive probe bias. Figure 4 shows a more systematic overview of the dependence of $U_M(U_{LP})$ on probe size, still for the reference case. For a big probe with 250 mm radius, the barrier would practically disappear ($U_{LP} - U_M \ll k_0 T_e/e$) for $U_{LP} - U_{pl}$ above a few volts. Such a disappearance of the barrier means that the probe becomes exposed to the ambient plasma and, as in region I, the probe characteristics should approach that of a free probe. Let us call the range of high positive potentials in which this happens region III and define the potential $U_2$ as its beginning. The situation can thus be summarized as follows (see Fig. 5): below the local potential in the sheath $U_1$, and also above a potential $U_2$ that depends on probe size, one can expect the standard OML theory of Eq. (1) to hold at least approximately. In a transition region II between these, one can expect the current to be reduced below the OML value. In this transition region, there is an upper limit given by OML theory and an approximate lower limit corresponding to a very small probe. The characteristics in the extreme small probe limit consist of only regions I and II, and the whole sweep is close to OML theory for a probe sampling the local parameters $U_1$, $n_{e1}$, and $T_e$ at the probe position (only below $U_1$ does this give the same functional dependence as a probe sampling the ambient $U_{pl}$, $n_{e0}$, and $T_e$). For such a probe sweep, the ambient $U_{pl}$ and $n_{e0}$ cannot be obtained without additional modeling. Finally, for finite-size probes, the shape of the curve in the transition region II depends on probe size in a way that departs from OML theory. A comparison between Figs. 5(b) and 1(b) shows that this could be a problem. In this region, two out of three key features for evaluation of plasma parameters are found, namely the slope and the position of the knee.

**IV. A SIMPLE ANALYTICAL PROBE-IN-SHEATH MODEL**

We here seek the ambient electron contribution to the probe current in region II as a function of the applied bias relative to the spacecraft, $I_e(U_{LP} - U_{SC})$. Let us make a thought experiment where we follow the time evolution of a situation where there are initially no electrons in the region enclosed by the thick curve in Fig. 2(b). Along this curve (or surface, in three dimensions), the potential is the same as that of the potential barrier at the $X$-point and this is the most negative potential that completely encloses the probe. We will call this surface the $X$-surface. Consider replacing the real probe with a point charge, placed at the probe center and of strength such that the potential outside the real probe’s surface is unchanged. The inflow of electrons from the ambient plasma through the $X$-surface will be the same as it was with the real probe. Let us call this electron current $I_{e, in}$. The electrons that have crossed the barrier will move about inside the potential trap around the point charge, but since it absorbs no electrons, none will be lost. There will eventually be an equilibrium in electron flow, where an equal number of electrons exit to the ambient plasma through the $X$-surface, i.e., $I_{e, out} = I_{e, in}$. Consider now, in this steady state situation, the electron density at the $X$-surface. All electrons at the $X$-surface have followed dynamical trajectories from the am-
bient plasma. According to Liouville’s theorem, the phase space density is constant along such trajectories. At equilibrium, we therefore propose that the electron density at the X-surface is the ambient plasma density reduced by a Boltzmann factor,

\[ n_e \propto e^{-\varepsilon(U_M - U_{pl})/k_BT_e}. \]  

(6)

The next step in this thought experiment is to replace the point charge with a probe that is so small that it collects a negligible current in the sense \( I_{pl} \ll I_{in} \). The collection of such a small current should have negligible influence on the equilibrium state. The X-surface, with an electron density given by Eq. (6), can then be regarded as a source region for the electron current drawn by this small probe.

Finally, we switch to a real probe with finite radius, and make two admittedly crude approximations in order to formulate a model intended to capture the qualitative features of the probe-in-sheath effects. Our method is a recipe in five steps for calculating the electron current \( I_e(U_{LP}) \) to a probe, for a given satellite-and-probe geometry, and assumed ambient plasma \( U_{pl}, n_{e0}, \) and \( T_e \):

1. Choose the real probe radius \( r_{LP} \) and boom length \( \ell_{boom} \), and a suitable value of the spacecraft radius \( r_{SC} \).
2. Choose the spacecraft floating potential as \( U_{SC} = -K_{float} k_BT_e/e \). The constant \( K_{float} \) has to be separately assessed, including, for example, photoelectron current and ion current (see Sec. V).
3. Obtain \( \lambda_{D} \) from \( n_{e0} \) and \( T_e \), and use Eq. (5) to obtain the potential \( U_1 \) at the probe position in the sheath.
4. For \( U_{LP} > U_1 \), obtain the potential \( U_M \) as a function of \( U_{LP} \) from calculated radial profiles such as in Fig. 2(e).
5. The probe-in-sheath model is obtained by modifying the OML equations so that the attractive-probe region begins at \( U_1 \) instead of \( U_{pl} \). In this attractive region the source plasma density is \( n_e \propto e^{-\varepsilon(U_M - U_{pl})/k_BT_e} \) and the probe potential is compared to the X-surface potential \( U_M \),

\[
I_e(U_{LP}) = \sqrt{8 \pi e \mu_{eff}} \frac{k_BT_e}{m_e} e^{-(U_{LP} - U_{pl})/(k_BT_e)} \\
\quad \text{when } U_{LP} < U_1,
\]

\[
I_e(U_{LP}) = \sqrt{8 \pi e \mu_{eff}} e^{-(U_{LP} - U_{pl})/(k_BT_e)} \sqrt{\frac{k_BT_e}{m_e}} \\
\quad \times \left( 1 + \frac{e(U_{LP} - U_M)}{k_BT_e} \right) \text{ when } U_{LP} > U_1.
\]  

(7)

As seen in Eq. (7), for \( U_{LP} < U_1 \), the usual OML model is retrieved. For \( U_{LP} > U_1 \) on the other hand, the proposed model deviates from OML in that it compares the probe potential to \( U_M \), rather than to the background plasma potential, and in that it rescales the density with a Boltzmann factor \( e^{-(U_M - U_{pl})/(k_BT_e)} \). Furthermore, the potential minimum \( U_M \), and therefore also the density rescaling, are themselves functions of the probe potential. Thus, when the probe is in the sheath [approximately when \( |U_1 - U_{pl}|/(k_BT_e/e) > 0.2 \)], the \( I_e(U_{LP}) \) curves according to this model are found to depend, through \( U_M \), on all geometry parameters \( \lambda_D, r_{SC}, r_{LP}, \) and \( \ell_{boom} \). These dependencies need to be understood and decoupled from the influence of the ambient parameters \( U_{pl}, n_{e0}, \) and \( T_e \), that are to be measured. The dependence on \( r_{LP} \) is the most significant and illustrated in Fig. 6 by a set of \( dU/dU_{LP} \) curves that are obtained from Eq. (7) for the reference case plasma parameters, and with varied probe radius (the same values of \( r_{LP} \) as in Fig. 4). These show all the features proposed based on general arguments in Fig. 2. The extent of the transition region and the curve shape within it depends on all the geometry factors and departs fundamentally from OML theory in several respects: (1) although the ambient plasma potential \( U_{pl} \) falls here, there is no knee or other obvious way to identify it, (2) there is no exponential part of \( I_e(U_{LP}) \) that can be used to obtain \( T_e \), and (3) the curve shape depends on the probe size \( r_{LP} \). In particular, the slope \( dU/dU_{LP} \), which is essential for the estimate of \( T_e \), decreases with decreasing probe size \( r_{LP} \). Standard OML analysis, by fitting an exponential to a part of measured \( I_e(U_{LP}) \) curves, could therefore overestimate the electron temperature.

V. COMPARISON WITH CASSINI SPACECRAFT DATA

For comparison with real data we have selected an often occurring sweep type as measured by the Langmuir probe\(^4\) on the Cassini spacecraft. This particular sweep was taken at 14 July 2005, 20:09:18, from the inner magnetosphere (plasma disk) of Saturn,\(^5\) where the plasma density is high so that the electron saturation current exceeds the photoemission current, and the probe acquires a negative floating po-
tential. We need to consider what populations, other than ambient electrons, contribute to the information-carrying derivative \( dI/dU_{\text{LP}} \) of the probe current (see Fig. 7). (1) There is a directed flow of ions due to the spacecraft motion (15–20 km/s) and the corotation of the plasma with Saturn (~40 km/s). This population gives a current that varies very little with probe bias, and therefore should contribute only marginally to the derivative. (2) There is a photoemission current from the probe, known to be of the order of 400 pA,\(^7\) which flows only when the probe is negatively charged or slightly positive. This current should therefore give a peak in the derivative at and closely above the potential \( U_1 \), with a width of a few eV, corresponding to the typical photoelectron energy \( k_B T_{\text{ph}}/e \). (3) The probe might also collect photoelectrons from sunlit parts of the spacecraft. Here, it is useful to introduce the concept of a photoelectron footpoint (see Fig. 7). This is the area from which photoelectrons would hit an unbiased probe, i.e., at \( U_{\text{LP}}=U_1 \). The current from this area is efficiently suppressed only when \( U_{\text{LP}} < U_{\text{SC}}-k_B T_{\text{ph}}/e \). This current grows with increasing probe potential in the whole range up to \( U_{\text{LP}}=U_1 \), giving a positive contribution to \( dI/dU_{\text{LP}} \). The shape of \( dI/dU_{\text{LP}} \) below and around \( U_1 \) therefore depends crucially on spacecraft geometry. Above \( U_1 \), the probe attracts and collects also electrons from an increasing area outside the photoelectron footpoint. A local structure that is well exposed to sunlight, for example, a part of the probe shaft, could give a large contribution to \( dI/dU_{\text{LP}} \). If such a structure is inside the photoelectron footpoint, this contribution is found below \( U_1 \) but if it is outside, the contribution is found somewhere above \( U_1 \). Figure 8 shows that the geometry is very complicated and predicting the photoelectron contribution would be difficult.

In summary, the ion current would not show up significantly on the measured \( dI/dU_{\text{LP}} \) curve, but we can expect a feature just above \( U_1 \) due to photoelectrons leaving the probe and also spacecraft photoelectron features with an unknown shape that depends on the geometry and the angle of illumination.

For the spacecraft floating potential we use

\[
U_{\text{SC}} - U_{\text{pl}} \approx -2.5 \frac{k_B T_e}{e}. \quad (8)
\]

This corresponds to \( k_{\text{real}}=2.5 \). This value is based on a particle-in-cell (PIC) simulation using a more realistic Cassini spacecraft geometry by Nilsson\(^9\) and holds approximately in a range of plasma densities \( 10^8 \text{ m}^{-3} < n_{\text{el}} < 10^8 \text{ m}^{-3} \) and temperatures 0.1 eV < \( k_B T_e < 10 \text{ eV} \). It also agrees with a calculation of our own of the floating potential of the Cassini Langmuir probe in the streaming-plasma environment of Saturn’s magnetosphere. We regard this value to be uncertain with about 20%, and sufficiently accurate for the present study.

We also need to choose an effective spacecraft radius \( r_{\text{SC}} \) for our probe-in-sheath model, such that the probe is at a correct depth, i.e., potential \( U_1 \), with Eq. (8) used for the spacecraft floating potential. Each curve in Fig. 9(a) represents a specific ratio \( \ell_{\text{boom}}/r_{\text{LP}} \). The thick (red) curve corresponds to the real Cassini spacecraft values \( r_{\text{LP}}=25 \text{ mm} \) and \( \ell_{\text{boom}}=1.5 \text{ m} \). We choose \( r_{\text{SC}}=8 \text{ m} \), for which the potential \( U_1 \) in Fig. 9(a) agrees with the PIC simulations of Nilsson,\(^9\)
using the same plasma parameters. This radius is clearly larger than the actual size of Cassini, but this is expected due to the difference in shape between the actual spacecraft and our spherical model. As seen in Fig. 8, the Langmuir probe does not reside outside a spherical, or even convex, surface, but is to some extent surrounded by the disc antenna and other parts. The spacecraft potential will therefore have a greater influence over the potential at the probe position than in a model with a spherical spacecraft of similar size. To compensate for this, the spacecraft radius in the model needs to be larger. With such fixed $r_{LP}$, $\ell_{boom}$ and $r_{SC}$, the thick (red) curve in Fig. 9(a) can be interpreted as the variation of $U_1$ with $\lambda_D$, while Fig. 9(b) shows the corresponding Boltzmann factor reduction of plasma density at the probe location. The dots on the curves show our reference case, with typical parameters from the Saturn E-ring environment. The density reduction at the probe is around 50% in the reference case, and varies considerably with $\lambda_D$, directly demonstrating the need to account for probe-in-sheath effects.

Before proceeding to the comparison between the OML and probe-in-sheath models and the data, we want to make a comment on fudge factors. These are parameters that can be embedded in a model and adjusted at will to make it fit better. The OML equations of Eq. (1) contain no fudge factor. There is no freedom of modification except by varying the plasma parameters $T_e$, $n_{e0}$, and $U_{pl}$. We claim the same to be true also for our probe-in-sheath model. All extra input parameters in Eq. (7), besides $T_e$, $n_{e0}$, and $U_{pl}$, can in principle be determined objectively: $\ell_{boom}$ and $r_{LP}$ are the real values, while $K_{float}$ and $r_{SC}$ can be based on separate investigations, in our case, the PIC model of the potential structure around Cassini.

FIG. 9. (Color online) A set of curves for assessing the influence of probe-in-sheath effects on a given spacecraft, in a situation where the plasma parameters are approximately known. (a) The potential $U_1$, as a function of normalized parameters $\lambda_D/r_{SC}$ and $\ell_{boom}/r_{SC}$, assuming $K_{float}=2.5$. (b) The resulting reduction of electron density by a Boltzmann factor at the probe position. The dots on the thick curves mark the Cassini standard case of Table I. In this parameter regime, the probe-in-sheath effects are clearly very sensitive also to variations in the plasma density and temperature, through the Debye length $\lambda_D$. Curve fitting with the probe-in-sheath model must therefore include a separate calculation of $U_{MK}$ for each tried combination of $n_e$ and $T_e$.

FIG. 10. A comparison between Cassini Langmuir probe data and the OML and probe-in-sheath models. (a) A measured probe sweep and its derivative. We propose that the feature found where $U_{LP}-U_{SC}$ is between 2.5 and 3.5 V might be photoelectron-induced. There is no clear knee in the derivative that directly gives $U_{pl}$ and no clear exponential part that directly gives $T_e$. Most useful feature is therefore the saturation level of the derivative that gives a value of the combined parameter $n_{e0}/(kT_e)^{3/2}$. (c) OML and probe-in-sheath model curves of $dI/dU_{LP}$ with $n_{e0}=5 \times 10^7$ m$^{-3}$, $kT_e=1.11$ eV, and $U_{SC}-U_{pl}=-2.5kT_e/e$. The vertical line indicates where the probe is at the plasma potential and “$U_1$” where it is at the potential of its immediate surroundings. (d) The same with $n_{e0}=7 \times 10^7$ m$^{-3}$ and $kT_e=2.17$ eV. (e) The same with $n_{e0}=9 \times 10^7$ m$^{-3}$ and $kT_e=3.59$ eV. We consider the best fit of the six model curves to be the probe-in-sheath model in panel (d). That also corresponds to the best agreement with an independent measurement of the plasma density $n_{e0}=(6.8 \pm 0.2) \times 10^7$ m$^{-3}$ by the upper hybrid probe. We regard this agreement rather fortuitous, considering the rough approximations we have made here.

Figures 10(a) and 10(b) show the current and its derivative, from the sweep by the Langmuir probe on Cassini, as a function of the applied bias voltage relative to the spacecraft. It has several common features. In particular, a “bump” on the $dI/dU_{LP}$ curve around 2–4 V bias is found on many probe sweeps from this region. The integrated area under this bump is generally too large for it to be due to the $\sim 400$ pA photomission current from the probe itself. A second feature found on all sweeps is a saturation in the $dI/dU_{LP}$ curve, often followed by a small decrease at high positive potential.
The saturation level varies with distance from the ring plane. Both in the OML model of Eq. (1) and in our probe-in-sheath model of Eq. (7), such a saturation level locks the combined parameter \( n_{e0}/\sqrt{T_e} \) and since \( U_{SC} - U_{LP} \) is determined by \( T_e \) through Eq. (8), this leaves only one degree of freedom in fixing the triple \( (U_{SC} - U_{LP}, n_{e0}, T_e) \). Consequently, a value of one of these three parameters determines the other two.

Each of Figs. 10(c)–10(e) show one OML and one probe-in-sheath model curve for a combination of electron temperature and plasma density that is consistent with the saturation plateau of the measured sweep. Out of these six curves, we regard the best fit (disregarding the proposed photoemission feature) to be obtained by the probe-in-sheath model and for the combination \( n_{e0} = 7 \times 10^7 \, \text{m}^{-3} \) and \( k_B T_e = 2.2 \, \text{eV} \). This is also the reference case used earlier in this paper. Separate support for this to be the correct combination is found in the independent plasma density obtained from the upper hybrid frequency \( f_{uh} = (f_{pe}^2 + f_{ce}^2)^{1/2} \) with \( n_{e0} = (6.8 \pm 0.2) \times 10^7 \, \text{m}^{-3} \). We therefore tentatively interpret the data in Figs. 10(a) and 10(b) as influenced by probe-in-sheath effects on photoemitted electrons from the spacecraft. Region I, where it might have been possible to estimate \( T_e \) from the exponential OML part (see, for example, Fig. 6), is corrupted by photoemission and in region II, a straightforward OML interpretation would be corrupted by probe-in-sheath effects.

VI. SUMMARY AND DISCUSSION

The main finding in this work is qualitative, rather than quantitative: the existence of the transition region, in which the probe characteristic is likely to depart from usual OML theory in respects that has a detrimental effect on the process of extracting plasma parameters from measured \( I(U_{LP}) \) by usual techniques: (1) the ambient plasma potential \( U_{LP} \) falls here, but there is no obvious knee or other feature to identify it, (2) there is in the transition region no exponential part of \( I(U_{LP}) \) that can be used to obtain \( T_e \), instead (3) the curve shape depends on the probe size in a way that, for reliable quantitative evaluation, needs to be separated from the dependencies on \( n_{e0} \) and \( T_e \). For more accurate results, models are needed that include a realistic, self-consistently obtained potential structure around the spacecraft, how it is modified by a variable probe potential, and better understanding (probably through particle simulations) of the electron collection to the probe in such a potential structure.

ACKNOWLEDGMENTS

H.G. is supported by the Belgian Science Policy Office through the Solar-Terrestrial Centre of Excellence.

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7. M. Holmberg (Exam work report, Uppsala University, Uppsala, Sweden, 2010).