Suprathermal Kappa populations in the solar wind: from observational evidences to realistic modelling

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Abstract

Suprathermal populations are ubiquitous in the solar wind, indicating plasma states out of thermal equilibrium, and an excess of free energy expected to enhance the kinetic instabilities. The generally accepted representation of Kappa distributions usually invoked to describe these populations in space plasma physics allows for two different alternatives, namely assuming the temperature either dependent or independent on the $\kappa$-index of the distribution. Here we demonstrate that only a $\kappa$-dependent temperature confirms the expectations providing a systematic stimulation of the temperature anisotropy instabilities, and present observational evidences in the favor of this approach.

The velocity distributions of plasma particles measured in the solar wind exhibit suprathermal tails which cannot be reproduced by the standard Maxwellian distribution functions. Instead, the Kappa power-laws describe these observations with a good accuracy, since a Kappa distribution function is nearly Maxwellian at low energies (thermal core) and decreases as a power-law at high energies, reproducing the high-energy tails of the observed distributions. Enhanced by the suprathermal (halo) populations, these tails indicate plasma states out of thermal equilibrium, and, implicitly, an excess of free energy expected to stimulate the kinetic instabilities of plasma waves. However, the existing Kappa approaches aiming to disclose the effects of these populations on the electromagnetic instabilities do not always confirm this expectation, but mainly show an inhibition of these instabilities in the presence of suprathermals.

The effects introduced by the suprathermal populations can be unveiled by means of a comparative analysis, contrasting the properties of the observed Kappa-distributed plasma, which incorporates both the core and halo populations, with those obtained only for the Maxwellian core of the distribution. Current methods adopted to describe the effects of suprathermal populations compare a Kappa distribution

$$F_\kappa(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \frac{\Gamma(\kappa + 1)}{(\kappa - 3/2)^{3/2} \Gamma(\kappa - 1/2)} \left[1 + \frac{m v^2}{2 k_B (\kappa - 3/2) T}\right]^{-\kappa - 1}$$

(1)
with a Maxwellian limit \((\kappa \to \infty)\) of same temperature \(T\)

\[
F_M(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right) \tag{2}
\]

where \(T\) is defined by the second order moment of the distribution

\[
T = \frac{1}{\kappa} \int d^3v v^2 F(v) \tag{3}
\]

with \(T_M = (1 - \frac{3}{2\kappa})T < T\) to be the core (Maxwellian) temperature \(T_M = T_{\text{core}}\), lower than \(T\). Fig. 2 shows that such a Maxwellian limit may approach the cooler core of the distribution. In this case the temperature of the global Kappa appears to be dependent on the power-index \(\kappa\)

\[
T = \frac{2\kappa}{2\kappa - 3} T_M > T_M. \tag{4}
\]

Comparison can be made by contrasting to the core of a given temperature \(T_M\). For lower values of \(\kappa\) the tails are enhanced, meaning additional suprathermal populations in the distribution, and the global temperature \(T\) will naturally increase with respect to the core temperature \(T_M\). Apparently, a \(\kappa\)-dependent temperature contradicts recent statements [5], which claim for Kappa distributed plasmas to have a \(\kappa\)-independent temperature, invoking classical thermodynamic principles demonstrated for standard Maxwellian systems, but still under debate for nonequilibrium plasmas. In our opinion, the temperature becomes automatically \(\kappa\)-dependent if it is defined as second order moment of the Kappa distribution.
Both these two assumptions are invoked in studies of the kinetic instabilities, but only for a $\kappa$-dependent temperature the instabilities show a systematic stimulation in the presence of suprathermals (i.e., for low values of $\kappa$). Relevant are the results obtained for the electromagnetic instabilities driven by temperature anisotropy $T_\perp \neq T_\parallel$ (with $\perp$ and $\parallel$ denoting directions relative to the magnetic field), i.e., for the cyclotron instabilities (driven by $T_\perp > T_\parallel$) in Ref. [2], and for the firehose instabilities (driven by $T_\parallel > T_\perp$) in Ref. [6]. As an example, Fig. 3 displays the case of the electron cyclotron (whistler) instability: the anisotropy thresholds (left panels) and the unstable modes (middle and right panels), wave-frequencies (top) and growth rates (bottom) for two distinct conditions. In all these panels, the instability is systematically stimulated only with respect to the Maxwellian core of lower temperature.

![Figure 3: Whistler instability: thresholds (left), frequencies and growth rates (middle and right panels)](image)

To improve comparison with the Maxwellian core further refinements are possible by making its peak to coincide with that of the global Kappa. To do that, we have to introduce the number densities of plasma particles, $n$ (total density) for the global Kappa, and $n_M$ for the Maxwellian core. The distribution functions discussed above will become: $F'_\kappa(v) = nF_\kappa(v)$ to describe the global distribution, and $F'_M(v) = n_MF_M(v)$ to reproduce the core. These are displayed in Fig. 4, where we can easily observe the progress made with these two new representations, especially for reproducing the core of the distribution. From $F'_\kappa(0) = F'_M(0)$ we find

$$n_M = n \frac{\Gamma(\kappa + 1)}{\kappa^{3/2}\Gamma(\kappa - 1/2)} < n$$

(5)

that means the number density of the core is naturally lower than the total number density. The number of particles in the global distribution is obviously higher than that in the core, but the volume unit used to calculate their number densities is the same.
We can conclude claiming that a global Kappa model which incorporates both the thermal core and the suprathermal (halo) tails of the observed distributions may be used to describe the effects of the suprathermal populations. To disclose these effects, comparison must be made with a Maxwellian limit that reproduces the core of the distribution, and here we have shown a straightforward method to determine such an approximating limit of the Kappa distribution function. In this case both the temperature and the number of particles in the global Kappa are naturally higher than for the Maxwellian core. The temperature of the Kappa distribution becomes $\kappa$-dependent, and this result is sustained by the predictions made for the kinetic instabilities, found in this case to be systematically stimulated by the suprathermals. In addition, a recent observational report [4] based on Helios, Cluster and Ulysses data indicates the same $\kappa$-dependency for the temperature of suprathermal populations, measured at different heliographic coordinates in the solar wind. Fig. 5 displays these measurements showing the Kappa temperature components (in parallel and perpendicular directions) increasing with decreasing the power-index $\kappa$.

References

Figure 4: Kappa distribution vs. Maxwellian limit of lower $T_M$ and lower $n_M$

Figure 5: Kappa temperature in the solar wind as a function of $\kappa$-index (after Ref.[4]).