ANALYSIS OF ELECTRON DENSITY PROFILES IN THE VICINITY OF foF2

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INTRODUCTION.

The amount of ionospheric electron density profiles available grows rapidly, particularly since many digital ionosondes have been installed. Today the University of Massachusetts Lowell network, where ARTIST profiles are computed from DIGISONDE records, is the main source of such informations.

In the present analysis, the Millstone Hill virtual heights during July 1989 in the range foF2 to foF2-1.2 MHz are compared with the corresponding virtual heights computed from the corresponding ARTIST profiles.

Possible improvements of the method are suggested.

METHOD OF ANALYSIS.

Huang and Reinisch (1982) (see also Budden, 1961) use the parameter \( g = \ln(f/f_5)/\ln(f_i/f_s) \) in the ARTIST procedure thus taking directly account of two important experimental parameters: \( f_5 \) the critical (top) frequency and \( f_i \) the lowest plasma frequency of the layer.

A second important choice mainly concerns the vicinity of the top frequency for which \( g = 0 \). The analytical representation of the height profile is:

\[
 h(f) = h(f_5) + g P(g) 
\]

the root ensures that the slope of the profile becomes infinite for \( g = 0 \) at the critical frequency as it must be. In general, \( P(g) \) is a fourth degree polynomial constructed as a linear combination of shifted CHEBYCHEFF polynomials, the coefficients of the combination being computed by a least squares procedure.

When the profile is known, \( h'(f) \) the virtual height corresponding to the sounding frequency \( f \) can be computed using

\[
 h'(f) = h_1'(f) + \int f \mu' df + \int g \mu' dg 
\]

\( h_1'(f) \) representing the contribution of underlying ionisation. This contribution will here be considered as a constant.

The basic data provided by the University of Lowell are hourly ionograms from Millstone Hill (\( f_5 = 1.4 \text{ MHz}, \Theta = 72.9^\circ \)) for July 1989 (SS = 127, 10.7 = 188). They give for each ionogram, the frequencies \( f_5 \), \( f_s = \text{foF2} \), the coefficients of the CHEBYCHEFF polynomials, and the values of \( h_0'(f) \) for every multiple of 100 kHz.

For each ionogram, the twelve values of \( h'(f) \) nearest to foF2 are compared with the corresponding \( h_0' \). An ad hoc value of \( h_1'(f) \) is determined so that the average of the twelve \( h_0'(f) \) is equal to that of the observed \( h_0' \).

RESULTS.

The main parameter used in the presentation and the discussion of the results is the separation \( \Delta \) between the critical frequency \( \text{foF2} = f_5 \) and the 12 frequencies considered which varies from ionogram to ionogram. Therefore, the study shall be restricted to the monthly mean of the hourly results.

Since the frequency step of the sounder is 100 kHz, the monthly hourly mean frequency separation \( \langle \Delta \rangle \) between foF2 and the highest frequency observed will approximately be equal to 50 kHz. Therefore, for the selected frequencies, the mean separations \( \langle \Delta \rangle \) will be 50, 150, ..., 1150 kHz.

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For each mean separation $\Phi$, the monthly hourly mean values $\langle h'_c - h'_0 \rangle$ and the corresponding dispersions $<\sigma>$ are computed from the individual differences $h'_c - h'_0$.

Since the ARTIST method involves a least squares fitting process, one should expect that the variations with $\Phi$ of the monthly mean differences should not be correlated to the time.

Figure 1 shows for each sixth hour the variation of $\langle h'_c - h'_0 \rangle$ as a function of the mean separation $\Phi$ (crosses). The first cross at righthand corresponds to $\Phi = 30$ kHz.

Unexpectedly, the curves are very similar during daytime (around 1700 UT) and the same holds during nighttime (around 0400 UT).

Figure 2 shows mean nighttime (from 2300 to 0900 UT) and mean daytime curves (from 1200 to 2100 UT) of the differences $\langle h'_c - h'_0 \rangle$ and of their dispersions $<\sigma>$.

It appears clearly that, at $\Phi = 150$ kHz or $f = f_{f2} - 150$ kHz, the mean computed value $h'_c$ is always much lower than the mean measured value $h'_0$. This difference is between three and four times greater during nighttime than by day.

The reality of the second zero crossing appearing during daytime at small $\Phi$ may be doubted because our hypothesis of a constant delay in the underlying regions is not necessarily correct. In fact, if the characteristic shape of the retardation after $f_{f1}$ is taken into account, the curves for daytime and nighttime look very similar.

The mean dispersion curves are also similar, only the maximum at $\Phi = 150$ kHz is more important during night as during day.

**DISCUSSION.**

The systematic variations of the mean differences may have some fundamental meaning.

The particular behaviour when $\Phi$ is smaller than 350 kHz may depend on the precision with which the value of $f_{f2}$ is extrapolated. Therefore, another comparison has been made increasing the individual values of $f_{f2}$ by 5, 10, 20, 30, 40 and 50 kHz. The behaviour of the $\Phi'$ in Figure 1 shows that for an increase of $f_{f2}$ equal to 50 kHz, the variation of $\langle h'_c - h'_0 \rangle$ with respect to $\Phi$ becomes consistent at any time of the day. We conclude that a numerical determination of $f_{f2}$ should be preferred to an extrapolation by an operator.

The effect of $\Phi$ on the mean differences is much greater during the night than during the day. In fact it increases with the difference between $f_{f5}$ and $f_{f1}$ showing that it becomes more and more difficult to represent the shape of the layer the range of frequencies gets broader. This is due to two factors: first, the statistical importance of the range near the critical frequency is greatest when the frequency range and the width of the zone are equal and this importance decreases as the range increases; second, the analytical description (2) of a layer is especially well suited for the range near the top frequency but the least squares fit, which leads to the coefficients of the CHEBYCHEFF polynomials, takes the whole domain of frequencies into account with the result that a mathematical feedback is introduced between the base of the layer and its shape near $f_{f2}$.

It should therefore be advisable to consider the vicinity of $f_{f2}$ preferentially and aim at a more precise evaluation of $f_{f2}$.

We have tried, for nighttime, another representation of the profile in the vicinity of $f_{f2}$. This is

$$h(f) = h(f_c) + A \sqrt{g_c} \left(1 + \lambda g_c \right) \quad g_c = \text{Ln}(f/f_c)/\text{Ln}(f_5/f_c)$$

where: $f_c$ is a corrected critical frequency (greater than the last observed frequency), $\lambda$ should be such that the relative importance of the relevant influence be a minimum, $f_5 - f_1$ should be not less than 3 kHz ($f_5 = f_c$).

In more than 98% of all cases, an iteration procedure has lead to a unique set of parameters ($\lambda$ is normally less than 0.05).

Figure 3 shows the results of this approach: only two values of $\langle h'_c - h'_0 \rangle$ are now greater than 5 km, but some residual effect (in the reverse direction) still remains at $\Phi = 150$ kHz.

We have further tried to give different weights to the $h'_0$ near the critical frequency. This reduces the undesired effect and shows that a more sophisticated fitting process is needed.

Figures 4a & 4b, similar to Figure 2, show a comparison of the results. The scale on the right side in Figure 4a is 10 times greater than the scale on the left side. The weighting procedure reduces the maximum deviation from 6 km to 4 km and also reduces the dispersion at $\Phi = 150$ kHz (Figure 4b).

Figure 4c shows the mean values of $f_c$ and $f_{f5}$ and indicates in the central line the differences (in kHz) between $f_{f2}$ and $f_{f2}$. These are, as expected, not far from 50 kHz.

Finally a comparison has been made between the shapes of the profiles resulting from ARTIST and those from our new method. With the top frequencies $f_{f5}$ and $f_{f1}$ and for identical values of $\Phi$ the values of

$$\Delta h = \sqrt{g} P(g) - A \sqrt{g_c} \left(1 + \lambda g_c \right)$$

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$$\Delta h = \sqrt{g} P(g) - A \sqrt{g_c} \left(1 + \lambda g_c \right)$$
have been computed. Figure 5 shows the average $\langle D \rangle$ and the dispersion of those differences during the nighttime as a function of $\langle D \rangle$. It indicates that the profiles obtained are mainly steeper than those computed using ARTIST.

Further consideration of the problem has shown that for the same series of ionograms the shape of the whole F2 layer can be represented by

$$h(f) = Q(f) + A \sqrt{g (1 + \lambda g)}$$

where $Q(f)$ is a polynomial of the fourth order. The differences $hc' - ho'$ have been computed for the ten frequencies nearest to the minimum frequency, in the middle of the range, and nearest to the top frequency. Their mean values and dispersions are respectively equal to $-2.3 \pm 3.0$, $3.1 \pm 1.8$ and $-0.6 \pm 6.3$ km.

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Fig. 3. Variation of \(<h'c' - ho'>\) as a function of \(<\sigma'>\) in the present representation (with error bars).

Fig. 4a. Variation of \(<hc' - ho'>\) in function of \(<\sigma'>\) for ARTIST and for the present representation. Note that the leftside scale for ARTIST is 10 times greater than the rightside scale.

4b. Comparison of the dispersions of ARTIST (x) and of the present representation (o).

4c. Mean values of \(F_s\) and \(F_{sc}\) and, in the central line, mean differences between the ARTIST and the corrected critical frequencies.

Fig. 5. Mean differences \(<\Delta h>\) and dispersions \(<<\sigma'>>\) between ARTIST and the present profiles in the vicinity of \(f_0F_2\) in function of \(<\sigma'>\).