Evidence for climatic attractors

GRASSBERGER draws attention to some problems related to the estimate of attractor dimensions in the case of sparse data sets. As parts of this paper may appear to have a controversial character for the unaware reader, in reference to our previous work we believe that some comments are in order.

It is surprising that with a number of points as small as 230 for the V28–238 core, Grassberger goes to phase spaces of dimension up to 11. In our own work with ~500 data points we reluctantly went up to dimension six, and considered that beyond this value the results were not reliable. In particular, the estimate of the slopes becomes increasingly difficult—indeed in fact after seeing Grassberger’s paper we repeated our computations by taking only one out of two or out of three data points and found no significant trend toward saturation for embedding phase space dimension up to six. We believe therefore that the conclusions drawn by Grassberger with so few data points are subject to caution. In addition it seems unlikely that the V28–239 core can be used for dimension estimates. There are very few raw data points, considering that they span a time interval twice as long as the V28–238 core. Moreover, the measurements beyond 10^6 yr are less reliable (P. Pidoux, personal communication).

Dr Grassberger minimizes the role of time lag τ. This may be theoretically correct, but in practice many researchers have pointed out the importance of the choice of τ in the calculation of attractor dimensions. The argument appealing to correlation time is of course legitimate, but in many cases this time may be too small to be estimated. It is unfortunate that no specific references regarding these points are provided by the author.

Dr Grassberger also warns against spuriously small dimension estimates arising from smoothed data. It should be realized that in real world complex systems direct access to physical variables may be impossible. In geology in particular, the connection between data (which refer to depth along the core) and physical variables (like temperature or ice volume) is established by appealing to a model. Depending on the type of model (which describes, among others, the sedimentation mechanism) a resolution of for instance, 2,000 yr (our data) or 5,000 yr (Grassberger’s data) is achieved. In either case, however, an interpolation is involved, since the raw data need not be equidistant. We regard this as a fact of life. As all current work on palaeoclimatology is based on such data, we do not see why we should refrain from using them.

In summary, these issues show how careful one has to be in discussing the behaviour of real world complex systems. They also highlight the need for abundant, high quality geologic data. We iterate once more our conviction that climatic attractors do exist, a conviction that is strengthened by independent dimension estimates reported recently by Fraedrich and Saltzman and co-workers.

C. NICOLIS
Institut d’Aéronomie Spatiale de Belgique,
Avenue Circulaire, 3
1180 Bruxelles, Belgium

G. NICOLIS
Faculté des Sciences de l’Université Libre de Bruxelles,
Campus Plaine, CP 226,
Boulevard du Triomphe,
1050 Bruxelles, Belgium