The mechanisms of formation of the Plasmapause*

by

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RESUME. — Les champs électriques de la Magnétosphère jouent un rôle important dans la formation de la brusque dépression de la densité du plasma thermique (0.1 – 1 eV) découverte par Carpenter [1963], et observée depuis lors à l'aide de différentes techniques. Les deux théories différentes proposées pour la formation de cette surface de discontinuité sont passées en revue. Le premier de ces mécanismes est basé sur l'approximation Magnétohydrodynamique. Il y est considéré que la position de la plasmapause est déterminée par la dernière équipotentielle électrique fermée. Dans l'approximation quasi-magnétohydrodynamique la valeur finie de la conductibilité électrique de Pedersen a été prise en considération pour déterminer la position de la plasmapause associée aux modèles de champs électriques de McIlwain [1972, 1974]. En raison de la réduction de la valeur de la conductibilité électrique transversale dans l'ionosphère nocturne, le mécanisme d'échange des tubes de force magnétique permet d'épulcher la plasmasphère dans la région nocturne aux environs de minuit temps local. Le transport de petits éléments de plasma vers l'extérieur est engendré par les dérives dues à la courbure des lignes de force et du gradient du champ magnétique ainsi que par la force centrifuge qui devient prédominante au-delà d'une "surface de Roche". Plusieurs conséquences de ces deux alternatives sont discutées.

ABSTRACT. — The magnetospheric electric field plays a determining role in the formation of the sharp drop in the thermal (0.1 – 1 eV) plasma density distribution investigated by Carpenter [1963], and observed since then by many different techniques. The two physically different theories proposed for the formation of this field aligned boundary are reviewed. The first of these mechanisms is based on the MHD approximation; it is considered that the position of the steady state plasmapause is determined by the last closed equipotential surface of the convection-corotation electric field distribution. In the second quasi-MHD approximation the finite value of the integrated Pedersen conductivity has been taken into account to determine the plasmapause position associated with McIlwain's [1972, 1974] electric field models. As a consequence of the reduced transverse conductivities in the night side region, flux tube interchange motions can peel off the plasmasphere near midnight. The resulting outward transport of small-scale plasma elements is driven by gradient B and curvature drifts, as well as by the outward directed centrifugal force which becomes important beyond a "Roche Limit" surface. Several consequences of both of these alternative approaches are discussed.

1. Introduction

Since the discovery of the knee in the equatorial electron density profile by Carpenter [1963], it became clear that the magnetospheric electric field distribution plays an important role in the formation of the plasmapause. The first convection electric field models were proposed by Axford and Hines [1961] and by Dungey [1961]. An impressive series of other electric field models has since been published in the literature. All of these models agree on two points: firstly, corotation predominates near the Earth as a consequence of the rotation of the ionosphere with the angular speed of the neutral atmosphere; the consequence is that the associated equipotential surfaces have nearly circular equatorial sections for \( L < 2-3 \). Secondly, “viscous like interaction” of the solar wind at the magnetopause, or interplanetary magnetic field lines merging, induce a sunward convective flow at large radial distances in the magnetosphere; the consequence is that a dawn-dusk electric field distribution exists across the magnetotail. Except for these two general attributes, there is a great diversity in the proposed convection flow patterns and in the associated electric field models, especially in the intermediate region between \( L = 3 \) and \( L = 10 \), i.e. precisely where the plasmapause formation takes place.

The purpose of this review will not be to discuss all the plasmapause models deduced from the different electric field distributions found in the literature, but it will be restricted to the physical mechanisms which have been proposed to explain the formation of the sharp field aligned drop in the ionization observed by many different techniques near \( L = 4 \).

The first mechanism proposed by Nishida [1966] is based on the magnetohydrodynamic approximation.

2. The magnetohydrodynamic approximation: infinite conductivities

Figure 1c, taken from Nishida [1966] shows the convection streamlines resulting from the superposition of the solar wind and corotational induced flow patterns illustrated in figures 1a and 1b. In the magnetohydrodynamic (MHD) approximation these streamlines correspond also to the equatorial sections of electric equipotential surfaces.

Two types of streamlines are drawn; 1) near the Earth, the streamlines are closed and encircle the Earth; 2) beyond the dashed curve the streamlines pass through the magnetotail where the magnetic field lines are supposed to be “open” and connected to outer space. Any zero energy proton or electron, or any cold plasma element convected along one of the latter streamlines, will finally reach the magnetopause region of “open” field lines where it can escape into outer space. Therefore it is expected that the flux tubes are empty when they are convected backwards from the tail toward the Earth. Furthermore, since the flux tube filling by ionospheric evaporation is a rather slow process, the ionospheric plasma distribution will never reach the saturation level (or diffusive equilibrium) in these high latitude flux tubes is expected to remain at a rather low value.

On the other hand, the cold plasma convected along closed streamlines inside the dashed line of figure 1c remains trapped along magnetic field lines which are always closed. In this case, diffusive equilibrium can be reached in less than \( \sim 6 \) days, and the resulting equatorial density can attain a saturation level of the order of \( 300 \) cm\(^{-3} \) at \( L = 4 \).

Therefore, in the MHD approximation, a sharp drop in the cold plasma density is expected to be maintained along the dashed line which coincides with the equatorial section of the last closed equipotential surface. According to Nishida, this last closed equipotential defines the position of the plasmapause discovered by Carpenter [1963], and already detected earlier by Gringauz et al. [1961] with ion-traps on board Lunik I and II.
It can be seen that the shape of the plasmasphere inferred from the MHD theory of the plasmapause formation and from Nishida's electric field model, is more or less comparable to the asymmetric shape of the observed plasmapause [Carpenter, 1966]. An even better fit to Carpenter's position of the plasmapause is obtained with the electric field model built up by Brice [1967]. This "best estimate" electric field model is illustrated in Figure 2.

Fig. 2
Magnetospheric electric field model proposed by Brice [1967]; Equatorial section of equipotential surfaces of the electric field induced into the magnetosphere by the solar wind (a), and by the corotation of the ionosphere (b). The combined field (c) is a "best-estimate" determined to reproduce as closely as possible the plasmapause positions observed by Carpenter [1966].

3. Analytical models

Figure 3 shows the electric field model of Kavanagh et al. [1968]. In this analytical model the solar wind induced electric field is supposed to have a uniform intensity $E_0$ directed from dawn to dusk across the magnetosphere. The last closed equipotential defines a tear-drop shaped plasmapause, and is characterized by a stagnation point at

$$r_s = \left( \frac{\Omega_E M}{E_0} \right)^{1/2}$$

in the 1800 LT direction [Raspopov, 1969], ($\Omega_E$ and $M$ are the angular velocity and magnetic moment of the Earth). At this singular point the total electric field is zero, and the $E \times B/B^2$ convection velocity is therefore also equal to zero. Just inside the plasmapause in the bulge region, the angular velocity of a cold plasma element is therefore smaller than $\Omega_E$. This is illustrated in Figure 4 where the dots represent the successive positions of, for instance, a whistler duct separated by constant intervals of time (1 hour) [Chappell et al., 1970b]. Beyond the
stagnation point the whistler medium is expected to drift westwards in a frame of reference fixed with respect to the magnetosphere.

Although the tear-drop model approximates rather poorly the actual dimensions of the plasmapause at all local times, it qualitatively reproduces the observed bulge in the dusk sector, and has consequently been used to study the deformations of the plasmapause under time-changing conditions. Indeed, when the intensity of the dawn-dusk electric field $E_0$ decreases, the last closed equipotential shifts outwards, and the older plasmapause boundary will drift along a new streamline pattern. Grebowsky [1970] and Chen and Wolf [1972] have calculated the time dependent deformation of an earlier plasmapause boundary after one or several consecutive sudden decreases of $E_0$. Using this method they have shown that the plasmasphere bulge has a tendency to drift eastward when the level of magnetic activity decreases. Conversely, a westward motion of the bulge was found after a sudden increase of $E_0$ or $Kp$. These results have been compared with the dynamical behaviour of the bulge reported by Carpenter [1970].

These theoretical calculations have also shown that several days after a sudden decrease of $E_0$, an extended thin tail-like structure unwarps from the main body of the plasmasphere [Chen and Wolf, 1972]. Grebowsky [1971] has suggested that such an attached plasma tail could explain the secondary peak in the density profile observed with the OGO 3 satellite by Taylor et al. [1970] just outside the Plasmapause-Light-Ion-Trough [Taylor, 1972].

Although most of these dynamical calculations are based on the teardrop model, similar results can be found with other plasmapause models. However, in the framework of the magnetohydrodynamic approximation it seems more difficult to describe quantitatively detached plasma elements [Chappell, 1974].

Many other types of analytical electric field distributions have been proposed in the literature, emphasizing different aspects or different effects which ought to be taken into account in theoretical model calculations of the magnetospheric electric field [Raspopov, 1970; Vasyliunas, 1970, 1972; Volland, 1973; Rycroft, 1974].

4. Numerical models

Wolf [1970] calculated numerically the electric field and plasma convection by including the effects of ionospheric Pedersen and Hall conductivities. In this model, illustrated in Figure 5, viscous interaction or line merging near the nose of the magnetosphere determine the boundary conditions at the magnetopause and across the distant tail region, and a “best-guess” non-uniform ionospheric conductivity distribution is taken into account. The shaded area is enveloped by the last closed equipotential, and represents the plasmapause boundary according to the MHD theory for the formation of the plasmapause.

![Diagram of Magnetospheric Electric Field Model](image)

**Fig. 5**

*Magnetospheric electric field model taking into account finite transverse electric conductivities in the ionosphere; a “best guess” non-uniform ionospheric conductivity is adopted with ad-hoc boundary conditions at the magnetopause to compute the equatorial section of the equipotential surfaces. The shaded areas are enveloped by the last “closed” equipotential surfaces, and correspond to the plasmasphere in the MHD approximation [Wolf, 1970]*.

An even more complicated and different electric field structure arises when an ion-sheet is injected into the equatorial plane of the magnetosphere as described by Jaggi and Wolf [1973]. Such an electric field distribution is illustrated in Figure 6 taken from Wolf [1974].

Considering that these already complex numerical models include neither the effects of neutral winds...
and of particle precipitation, the effect of internal electric currents on the magnetic field model, nor the effect of the cold plasma population, it can be concluded, with Wolf [1974], that more complete theoretical models are presently beyond our grasp.

5. Magnetospheric electric fields deduced from observations

Electric fields have been measured by various technique in the ionosphere and in the magnetosphere [Mozer, 1973; Haerendel, 1973]. An interesting new method to deduce the distribution of magnetospheric electric fields from energetic particle flux measurements has been developed by McIlwain [1972]. Using the dynamic spectra obtained by De Forest and McIlwain [1971] on board the geostationary satellite ATS 5, it has been possible to infer the large scale electric field intensity that the 50 eV – 50 keV protons and electrons have experienced along their drift path between the time of their substorm injection and the time of their encounter with the ATS 5 satellite. The dashed lines in Figure 7 illustrate the equatorial sections of the equipotential surfaces corresponding to McIlwain’s [1972] E3 electric field model determined by this method. A revised version, E3A, was proposed later to incorporate certain low altitude satellite measurements; however, this E3A model has proved to be incompatible with the ATS 5 plasma data (McIlwain, 1973, personal communication). The latest version, E3H, quite similar to the E3 model, is illustrated in Figure 8. The equation of the equipotential lines (dashed) have usefully been represented by a numerical series of 6 x 20 exponential terms whose values have been tabulated by McIlwain [1972, 1974]. The E3H model has also proved to be useful to interpret S3 satellite measurements (Fritz, 1974, personal communication).

The remarkable feature of the E3 and E3H magnetospheric electric fields is that, unlike in the tear-drop model and in other theoretical models, there is no stagnation point in the dusk sector. Therefore the last closed equipotential surface extends to considerable distance in the dusk sector. Since in the MHD theory described in Section 2 the last equipotential coincides with the plasmasphere, the presumed plasmasphere (shaded area in Figures 7 and 8) deduced from the E3 and E3H models would eventually reach the magnetopause.
Magnetospheric electric field model E3H of McIlwain [1974]; the dashed curves represent the equatorial sections of equipotential surfaces. The last "closed" equipotential corresponds to the outer boundary of the light shaded area which represents the plasmasphere according to the MHD theory of the formation of the plasmapause.

The positions of the plasmapause deduced from different types of observations [Carpenter, 1966; Chappell et al., 1970a, 1971; Taylor et al., 1970, Brace and Theis, 1974] are illustrated in Figure 9. For comparison we have also plotted the position of the last equipotential curve deduced from the E3H model. The results illustrated in Figure 9 indicate that, if the plasmapause is an equipotential surface, it is different from the last closed equipotential surface of McIlwain’s model E3H. Another mechanism is therefore expected to be responsible for the formation of the plasmapause. In the following section we describe the mechanism proposed by Lemaire [1974].

6. The quasi-magnetohydrodynamic approximation: finite conductivities

The ionospheric plasma distribution along the magnetospheric field lines is determined by the gravitational potential, and, at large radial distance \( r > 4 - 5 R_E \), by the centrifugal potential (Koc-karts and Nicolet, 1963; Angerami and Thomas, 1964). Considering centred dipole magnetic field lines, one can define a "Roche Limit" surface for the ionospheric plasma as for rotating neutral atmospheres (see, Appendix A). This "Roche-Limit" (R.L.) surface is the locus of the points where the total gravitational and centrifugal potential distribution

\[
\phi_t = -\frac{GM}{r} - \frac{1}{2} (\Omega \times r)^2,
\]

has its maximum value along a field line. At this surface the components of the gravitational force and of the centrifugal force, along the magnetic field direction, balance each other. Inside the R.L. surface the ionospheric plasma tends to fall down (preferentially along the field lines), and piles up against the Earth according to hydrostatic equilibrium; outside the R.L. surface the 0.1 - 1 eV ionospheric plasma tends to move upwards due to the predominance of the centrifugal force.
The equatorial distance of this "Roche Limit" surface is given (in Earth radii) by
\[
L_e = \left( \frac{2 \, G \, M_E}{3 \, \Omega^2 \, R_E^2} \right)^{1/3}
\]
where \( M_E \) and \( R_E \) are the mass and radius of the Earth, and \( \Omega \) the angular speed of the plasma around the magnetic dipole axis. Along field lines with \( L \) smaller than \( L_e \), the total potential, \( \phi_e \), is a monotonically increasing function of altitude with a maximum at the equator (solid lines in figure 10). On the other hand, field lines with \( L > L_e \) have two symmetric maxima out of the equatorial plane and a minimum at the equator (dashed lines in figure 10).

Using McIlwain's [1974] electric and magnetic field models \( E3H \) and \( M2 \) it is possible to calculate the electric drift velocity, \( E \times B / B^2 \) (see, arrows in figure 11). From the azimuthal component of these convection velocities one can calculate the local angular velocity \( \Omega \) of zero-energy protons and electrons. Using this distribution of \( \Omega \) and equation (3), it is also possible to determine numerically the equatorial section of the "Roche Limit" surface shown in figure 12 by the solid line running across the magnetosphere from dawn to dusk.

The region I is bounded outwardly by the equipotential surface which is tangent to the R.L. surface at the point of innermost penetration of the R.L. surface (i.e. at \( L = 4.5 \) near 0100 LT); the outer shaded region III is bounded inwardly by the last closed equipotential surface of the \( E3H \) model; and finally, the intermediate region II (unshaded in figure 12).

The region I: In region I the equipotential surfaces are closed and they never intersect the R.L.
A flux tube which would have been depleted is gradually filled up during the day by ionospheric evaporation at a rate of about $3 \times 10^8 \text{cm}^{-2} \text{sec}^{-1}$ at 1000 km altitude [Park, 1970]. Partial depletion during night time at a rate of $1.5 \times 10^8 \text{cm}^{-2} \text{sec}^{-1}$ has also been observed by Park [1970], showing that ionospheric plasma along the closed field lines of region I is in a dynamic non-equilibrium state due to replenishment during the day hours and to nocturnal and magnetic storm time depletions [Chappell, 1972]. A net outward mass transport in the plasmasphere cannot be excluded owing to the net excess of upward ionization flux. Such a slow expansion of thermal plasma can also be inferred from the observed equatorial density profiles, $n_{eq}$, between $L = 3.5$ and the plasmapause [Chappell, 1972; Carpenter and Park, 1973]. Indeed, in this region, $n_{eq}$ is generally proportional to $L^{-4}$, and decreases more rapidly with $L$ than for diffusive equilibrium, i.e. when there is no net expansion of the thermal plasma.

**The region III**: In region III where the equipotential surfaces reach the magnetopause, the plasma is convected towards the “open” polar-cap magnetic field lines where it can escape into the extended magnetotail region as suggested by Nishida [1966]. Quantitative hydrodynamic and kinetic descriptions of these polar wind flows have been proposed, respectively, by Banks and Holzer [1968, 1969] and by Lemaire and Scherer [1970, 1973].

**The region II**: In the dayside part of the intermediate region II where the total potential has a single maximum in the equatorial plane, ionospheric evaporation will replenish steadily the convecting flux tubes [Chappell, et al., 1971; Corcuff, et al., 1972]. This means that more and more “ballistic” particles with pitch angles inside the loss cone are stored by collisional pitch angle diffusion on “trapped” orbits with pitch angles outside the loss cone. The small-angle-scattering of charged particles by Coulomb interactions tends to establish an isotropic and Maxwellian velocity distribution which is characteristic of the diffusive equilibrium density distribution [Lemaire and Scherer, 1974].

During this refilling process, the dayside flux tubes are convected nearly along equipotential surfaces toward the night side part of region II, beyond the “Roche Limit” surface. Here, a fraction of the particles (those with large equatorial pitch angles) become trapped in the potential well near the equatorial plane (see figure 10). These trapped particles are “disconnected” from the lower ionosphere and can move independently of those which bounce along the field lines with mirror points located in the collision-dominated region of the ionosphere.

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**Figure 12**

The “Roche Limit” surface for the E3H model of Mcllwain [1974]; the dashed lines are electrostatic equipotentials. The solid line is the equatorial section of the “Roche Limit” surface where the total potential, $\Phi_T$, has a maximum. The boundary of the inner shaded region I is tangent to the “Roche Limit” surface near midnight; the inner boundary of the outer shaded region III is the last “closed” electric equipotential surface. If the ionospheric plasma populating this region is in hydrostatic equilibrium, it can be shown (Appendix B) that the streamlines are parallel to the equipotential surfaces, and that neither outward nor inward mass transport results in this ideal equilibrium situation.

Furthermore, if the integrated transverse electric conductivity is infinite (MHD approximation), any fieldaligned plasma inhomogeneity (e.g. a whistler duct) will corotate with the local convection velocity given by eq. D1 (see Appendix D). However, if the integrated Pedersen conductivity, $\Sigma_P$, is finite, flux tube interchange motions can appear and move full and empty flux tubes toward levels where their density is equal to the background density [Brice, 1973]. The field lines of region I corresponding to the plasmasphere dip into the low latitude ionosphere where $\Sigma_P$ is rather high ($\Sigma_P > 6 \text{ mho}$, at local noon; Hanson, 1961, see also Wolf, 1974); therefore, the maximum radial velocity of flux tube interchange remains small ($v_{eq} < 0.01 \text{ L/hour at } L = 4$ near local noon; see eq. D3 in the Appendix D). Consequently, in region I, the plasma elements are expected to drift closely along equipotential surfaces as in the MHD approximation.
Furthermore, since in the midlatitude trough ionosphere the integrated Pedersen conductivity is significantly reduced (\( \Sigma_p \equiv 0.8 - 0.08 \) mho, Hanson, 1961; Wolf, 1974), the maximum flux tube interchange velocity increases drastically by more than an order of magnitude after sunset. It is shown in Appendix D that, beyond the "Roche Limit" surface, the flux tube interchange motion driven by an external force, \( F \), will work in the same direction as the interchange motion driven by gradient \( B \) and curvature drifts [Richmond, 1973]. Both of these driving mechanisms can transport full flux tubes outwards and the less dense flux tubes inwards with a maximum velocity, \( v_{eq} \simeq 0.5 \) km/hour. Although this outward drift velocity of small-scale plasma inhomogeneities is smaller than the dominant convection velocity \( (E \times B/B^2) \simeq 2 \) km sec\(^{-1}\) i.e. 1.2 earth radii/hour, at \( L = 4 \), it is large enough to transport those elements across region II whose minimum thickness is only 1 \( R_E \) in the post-midnight sector. Once into region III, those small-scale plasma elements will be convected towards the "open" field lines of the tail as described above, and will escape towards outer space as suggested by Nishida [1966].

From figure 12 it can be seen that the plasma shell just outside the equipotential surface tangent to the "Roche Limit", will be peeled off near midnight at the point of deepest penetration of the R.L. surface. It should be noted that Chappell et al. [1971] have deduced from OGO 5 observations that the formation of the plasmapause takes place before dawn. Indeed, the position of the plasmapause after 0600 LT appears to be determined by the level of magnetic activity present at an earlier time, when the plasma corotated through the midnight region.

It has also be found from whistler and polar orbiting satellite observations that midnight is the local time region where the plasmasphere responds most rapidly to increases in magnetic activity [Carpenter and Park, 1973]. When the level of magnetic activity increases, the convection electric field induced into the magnetosphere by the solar wind is supposed to be enhanced. As a consequence, the "Roche Limit" surface will penetrate deeper into the magnetosphere. Therefore a larger fraction of the plasmasphere could be peeled off near midnight. If this new electric field distribution lasts for at least 24 hours, a new steady state plasmapause can be formed at a smaller radial distance from the Earth.

If \( K_p \) decreases to a very low value, the solar wind induced convection electric field may decrease to a small value inside the magnetosphere. Under such extremely quiet time conditions the corotation electric field can remain predominant up to considerable radial distances, and the equatorial section of the "Roche Limit" then becomes a circle at \( L = 5.78 \) (see eq. 3, with \( \Omega = \Omega_E \)). After 6 or 7 days of low magnetic activity the region between the old plasmapause position and the new one will be refilled with ionospheric plasma. This replenishment during a magnetic recovery phase has been observed by Chappell et al. [1971], Corcuff et al. [1972], and Park [1973].

During this period of low \( K_p \), the old plasmapause boundary will experience dynamical deformations which can be followed numerically by the method described by Grebowsky [1970, 1971], and by Chen and Wolf [1972] in the case of the tear-drop model.

Note also that \( L_p = 5.7 \) is the approximate position of the observed plasmapause in the midnight-dawn sector when \( K_p \) is set equal to zero in Carpenter and Park's [1973] statistical relation between the plasmapause position \( (L_{pp}) \) and the \( K_p \) index

\[
L_{pp} = 5.7 - 0.47 K_p
\]

This compares well with the value of \( L_c = 5.78 \) obtained above for \( \Omega = \Omega_E \).

When magnetic activity fluctuates around some average value, the midnight peeling mechanism described here will cause indentations in the post-midnight plasmapause surface. Such longitudinal irregularities have actually been observed by Park and Carpenter [1970], and Bullough and Sagredo [1970].

The \( E3H \) electric field distribution of McIlwain [1974] corresponds to \( K_p = 1 - 2 \). The steady state plasmapause position deduced with this electric field model is illustrated in figure 13; here the shaded area corresponds to the statistical limits within which the plasmapause is found between 2 000 LT and 0500 LT, when \( K_p = 1 \). These statistical limits have been deduced by Rycroft and Burnell [1966]. The agreement is satisfactory.

7. Discussion

Figure 14 illustrates the three regions discussed in the previous section. The innermost shaded area corresponding to the steady state plasmasphere for \( K_p = 1 - 2 \) is bounded by the electric equipotential surface which is tangent to the "Roche Limit" surface near midnight. This region is filled with corotating cold plasma evaporated from the low latitude ionosphere. The density in this collision-dominated region has a rather high value and is distributed along the magnetic field lines according to a more or less isotropic and Maxwellian velocity distribution. However, as a consequence of the daytime upward ionization flow and the downward plasma flux during the night, slight departures from pitch angle isotropy and diffusive equilibrium are expected to be present in the outermost part of the plasmasphere.
of the very long filling time of the high latitude flux tubes and their repeated depletions when they are convected to the polar cap and magnetopause. Indeed, the plasma content of the “open” magnetopause field lines can be released to outer space according to the suggestion of Nishida [1966].

The intermediate region corresponds to the plasmatrough [Carpenter, 1966]; the magnetic field lines through it probably enter the Light-Ion-Trough ionosphere [Taylor and Walsh, 1972]. Kilovolt ring current particles are detected in the plasmatrough with fluxes and densities varying with time after magnetic storm injections [Frank, 1965]. The cold plasma is also in a non-equilibrium dynamic state. During the day the combined effect of slow ionospheric evaporation and Coulomb collisions stores $0.1 - 1.0$ eV protons and electrons on trapped orbits corresponding to large equatorial pitch angles. This results in a monotonically increase of the equatorial density as a function of local time. Since the evaporation flux of thermal ions from the topside ionosphere is not very dependent on latitude, the equatorial density distribution is expected to be approximately proportional to the inverse of the flux tube volume (i.e. $n_{eq} \propto L^{-4}$). In the dayside the flux tubes or the plasma they contain are convected along streamlines more or less parallel to the equipotential surfaces. However, when they reach the night side part of the plasmatrough region the integrated Pedersen conductivity decreases rapidly; the maximum velocity characterizing magnetic flux tube interchange motions becomes of the order of $0.5$ L/hour.

Beyond the “Roche Limit” surface where the outward directed centrifugal force is larger than the gravitational force, the denser plasma elements will move outwards across the narrow nocturnal plasmatrough region towards the region of “open” equipotential surfaces. Once into this outer region, the plasma escapes when it reaches the magnetopause boundary. This flux tube interchange motion driven by the centrifugal force as well as by the gradient $B$ and curvature drifts also leads to inward motion of the less dense plasma elements [see Appendix D].

The innermost shell of the plasmatrough is peeled from the plasmasphere near midnight where the formation of the plasmapause takes place, and where the plasmasphere responds most rapidly to changing magnetic activity levels. A sharp drop in the density of the thermal trapped particle population (i.e. with pitch angles outside the loss cone) is therefore expected to appear beyond midnight. The void of plasma created by this peeling off mechanism will also enhance the upward $H^+$ and $H^+$ at the newly formed plasmapause boundary. Such peaks in the upward light ion fluxes have actually been observed at the plasmapause with the ISIS II satellite by Hoffman [1974].
The stability of the strong density gradient at the plasmapause has been discussed by Richmond [1973] and Brice [1973] who showed that only a very small number density of energetic ring current particles is needed in the plasmatrough to maintain pressure balance across the plasmapause surface.

8. Conclusions

To conclude this review, it can be said that the model of Lemaire [1974] differs from the earlier MHD models by the existence of a transition plasmatrough region where dynamic processes such as ionospheric filling during the day and flux tube interchange motions during the night occur. In the MHD approximation based on the assumption that the integrated transverse electric conductivities are infinite, the formation of the plasmapause is determined by the presumed existence of a stagnation point in the duskside region. In the newer approach the plasmasphere appears to be peeled off near midnight as a consequence of the gradient B and curvature drifts and of the centrifugal force.

Future observations and theoretical studies will certainly help us to decide which of these two alternative theories for the formation of the plasmapause is the more realistic. Other aspects not discussed here (e.g. the detached plasma elements observed in the afternoon sector, the steepness of the density gradient at the plasmapause, etc.) have to be worked out quantitatively before a final description can be given.

Appendix
A. The Roche Limit in a neutral atmosphere

Let us consider an extended neutral atmosphere around a planet of mass $M$ rotating with an angular speed $\Omega$. The equilibrium pressure $p$ distribution of the gas, of density $\rho$ satisfies the equation

$$\rho (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla p + \rho \nabla \phi_g = j \times B \quad (A1)$$

In the case of uniform rotation with no radial expansion, $\mathbf{v} = \Omega \times \mathbf{r}$, and $(\mathbf{v} \cdot \nabla) \mathbf{v} = \nabla \phi_\Omega$, where $\phi_\Omega = -\frac{1}{2}(\mathbf{\Omega} \times \mathbf{r})^2$ is the centrifugal potential.

The pressure gradient is then balanced by the total gravitational and centrifugal forces,

$$\nabla p + \rho \nabla (\phi_g + \phi_\Omega) = 0. \quad (A2)$$

The equilibrium pressure has a minimum on a cylindrical surface called the Roche Limit. The equatorial radius of this surface where $\phi_g + \phi_\Omega$ is maximum is given by

$$r_{RL} = \left(\frac{GM}{\Omega^2}\right)^{1/3} \quad (A3)$$

When $M$ and $\Omega$ are the mass and angular speed of the Earth, $r_{RL} = 6.6$ Earth Radii.

Unless, beyond this distance, a positive pressure gradient can be maintained artificially (e.g. by a wall), the gas expands and the pressure decreases steadily to zero at infinity. The rate of expansion $(p \cdot \nabla) \mathbf{v}$, depends on the imbalance between the actual pressure gradient and the external force density (eq. A1).

B. Hydrodynamic expansion of an ionized atmosphere

If one considers an ionized atmosphere, the equation of motion of the plasma is given by

$$\rho (\mathbf{v} \cdot \nabla) \mathbf{v} + \nabla p + \rho \nabla \phi_g = j \times B \quad (B1)$$

where, under steady state conditions, the electric current density $j$ is related to the magnetic induction $B$ by the equation

$$4\pi j = \text{curl} B \quad (B2)$$

with the consequence that $\text{div} j = 0$.

If $B = 0$ or if $B$ is curlfree, as in most magnetic field models considered (e.g. the dipole model), $j$ should be zero and the equation of motion becomes the same as for a neutral gas (eq. A1), where $\mathbf{v}$, $\rho$ and $p$ are respectively the bulk velocity, the total density and kinetic pressure of the electrons and ions. It can be verified, from the more familiar form of equation (B1)

$$j_L = [\text{div} p + \rho \nabla \phi_g + \rho (\mathbf{v} \cdot \nabla) \mathbf{v}] \times B/B^2, \quad (B3)$$

that the electric current is indeed equal to zero when the plasma pressure tensor satisfies equation (A1).

However, if currents are allowed to flow inside the system (for instance carried by ring current particles) the induction field $B$ will no longer remain curlfree, and the electromagnetic $j \times B$ force will modify the equilibrium pressure distribution for which there would be no radial expansion of the whole plasma. In the case of uniform rotation without expansion, this equilibrium pressure satisfies the equation

$$\nabla \left(p_e + p_i + \frac{B^2}{8\pi}\right) - \frac{1}{4\pi} (B \cdot \nabla) B +$$

$$\rho \nabla (\phi_g + \phi_\Omega) = 0 \quad (B4)$$

where $B$ is the sum of an external magnetic field ($B_d$), and a non-curlfree induction ($B_e$) resulting from the
internal electric current distribution. When the kinetic and magnetic pressure tensors satisfy such an equilibrium equation (as it is generally assumed [Vasyliunas, 1970]), the streamlines of the plasma are closed loops and there is no net outward mass transport.

However, any reduction in the pressure gradient, or any reduction in the electric ring current density would result in an expansion of the whole plasma at a rate which depends on the imbalance between the actual pressure gradient and the gravitational, centrifugal and electromagnetic forces. In this case the streamlines of the plasma will become spirals as in the case of the solar corona-solar wind expansion. The expansion velocity \( v \) (which may remain small compared with the thermal speed of the particles) is then obtained by integrating equation (B1).

C. Diffusion between hot and cold plasma

The bulk velocity \( v \), the density \( \rho \), and the kinetic pressure \( P \), are quantities averaged over the total velocity distribution (including hot and cold particles). However, from particle flux measurements one usually determines \( v, \rho, p_1, p_2 \) in a limited range of particle energies, depending on the sensitivity of the instruments. Therefore, it is useful to define \( v_{\text{th}, e}, v_{\text{th}, p}, v_{R, C, e}, v_{R, C, p}, P_{R, C, e}, P_{R, C, p} \) etc. for the thermal electrons and protons, as well as for the energetic (ring current, \( R.C. \)) electrons and protons.

By definition one has

\[
\begin{align*}
\rho v &= m_e (n_{\text{th}, e} v_{\text{th}, e} + n_{R, C, e} v_{R, C, e}) + \\
&+ m_p (n_{\text{th}, p} v_{\text{th}, p} + n_{R, C, p} v_{R, C, p}) = \Sigma_i m_i n_i v_i \\
\rho &= \Sigma_i m_i n_i \\
p &= \Sigma_i p_i
\end{align*}
\]

(C1) (C2) (C3)

with the quasi-neutrality condition

\[
\Sigma_i Z_i n_i = n_{\text{th}, p} + n_{R, C, p} - n_{\text{th}, e} - n_{R, C, e} = 0
\]

(C4)

Therefore, outside the plasmapause near the equatorial plane where the hot \( R.C. \) particles and the cold ionospheric particles have comparable concentrations

\[
(n_{\text{th}, e} \approx n_{\text{th}, p} \approx 1 - 10 \text{ cm}^{-3}, n_{R, C, e} \approx n_{R, C, p} \approx 0.1 - 1 \text{ cm}^{-3})
\]

one cannot exclude the possibility that the average bulk velocity \( v \) is different from the individual velocities \( v_i \) of the thermal and energetic plasmas. Even in the ideal case when there is no global expansion (i.e. for closed streamlines), it is possible that the cold particles move outwards and that the ring current particles migrate inwards in such a way that there is no net outward mass transport (\( \oint \mathbf{v} \cdot d\mathbf{l} = 0 \) for any closed integration contour \( \mathbf{E} \)).

D. Interchange of flux tubes

For steady state conditions, electric equipotential surfaces can be defined to describe \( E_1 \), the electric field component perpendicular to \( B \). According to the generalized Ohm's law (Spitzer, 1956), when the integrated Pedersen conductivity \( \Sigma_p \) is supposed to be infinite (MHD approximation), the perpendicular component \( (v_\perp) \) of the bulk velocity is orthogonal to \( E \).

\[
v_\perp = \frac{F_x \times B}{B^2}
\]

(D.1)

Therefore the streamlines are parallel to the equipotential surfaces, as a consequence of the assumption that \( \Sigma_p = \infty \).

However, the actual integrated Pedersen ionospheric conductivity has a finite value with a minimum during night time hours. When \( \Sigma_p \ll \infty \), the streamlines are not necessarily parallel to the equipotential surfaces, and the flow velocity is determined by the momentum equation just as in a friction-dominated flow. As in general frictional systems, the equation of motion (B1) will determine the velocity rather than the acceleration [Dungey, 1968]. The characteristic time to reach the asymptotic flow regime, where \( \partial u/\partial t = 0 \), is inversely proportional to \( \Sigma_p \).

Furthermore when \( \Sigma_p \ll \infty \), magnetic flux tube interchange can occur under certain conditions described by Gold, [1959], Sonnerup and Laird [1963], Richmond [1973] and Brice [1973]. Longmire [1963, p. 75] has shown that a magnetic flux tube element, filled with cold plasma and surrounded by flux tubes which are "empty", moves in the direction of the external (gravitational) force \( F = m g \) (see fig. 15). If the integrated conductivity is assumed to be zero, the acceleration of the flux tube, and of the plasma contained in it, is equal to \( g \).

However, when the transverse conductivity \( \Sigma_p \) in the region between the magnetic field lines from the surface of the plasma element has a finite value, the electric potential appearing between the eastward and westward edge of the element will drive a current \( (J) \) into the ionosphere (see figure 15) with upward and downward field aligned currents closing the electrical circuit. Brice [1973] has shown that the plasma element will reach an asymptotic velocity when the rate of loss of gravitational potential energy is equal to the dissipation of energy by ionospheric Joule heating. He calculated that the equatorial velocity of the flux tube is limited by the asymptotic value

\[
v_{eq} = \frac{8 L N T m_i g_0}{B^2 \Sigma_p}
\]

(D2)
where \( N_T \) is the total flux tube concen per unit area in the ionosphere, \( B_1 \) the geomagnetic field in the ionosphere, \( m_i \) the ion mass, and \( g_\theta \) the gravitational acceleration at the Earth's surface. For \( L = 7 \), \( N_T = 3.10^{13} \) cm\(^{-2}\), \( B_1 = 0.6 \times 10^{-4} \) weber m\(^{-2}\), and \( \Sigma_p = 0.2 \) mho, the inward velocity in the equatorial plane is \( v_{eq} = 0.2 \) L/hour, corresponding to an equatorward velocity of 10 m sec\(^{-1}\) at the ionospheric level. It is obvious that beyond the \( \text{Roche Limit} \), where the external force \( F \) is directed outwards, this mechanism will work in the opposite direction, and will move the full flux tube outwards and the less dense flux tubes inwards.

Richmond [1973] showed that the gradient \( B \) and curvature drifts of particles of energy \( \epsilon \), contained in a field aligned plasma inhomogeneity, will also contribute to move the full tubes outwards and the empty flux tubes inwards. The asymptotic equatorial velocity associated with this driving process

is

\[
v_{eq} = \frac{256 \sin \theta}{105 \sum_p B_0^2} L^6 \left(4 - \frac{3}{L}\right)^{1/2}
\]

where \( n \) is the density of particles with average energy \( \langle \epsilon \rangle \), \( B_0 \) is the magnetic field in the ionosphere, and \( L \) is the magnetic dip angle. For

\[
L = 4, \quad n = 3 \times 10^2 \text{ cm}^{-3},
\]

\[
\langle \epsilon \rangle = 0.4 \text{ eV (i.e. } T \approx 5000 \text{ K),}
\]

\[
\Sigma_p / |\sin \theta| = 0.3 \text{ mho,}
\]

Richmond [1973] finds \( v_{eq} = 0.7 \) L/hour, corresponding to a poleward velocity of 75 m sec\(^{-1}\) at the ionospheric level.

Note that in the MHD approximation (when \( \Sigma_p = \infty \), the flux tubes will not be able to interchange, since according to equations (D2) and (D3), the maximum asymptotic velocity \( v_{eq} \) is zero. However, when the integrated Pedersen conductivity is finite beyond the \( \text{Roche Limit} \) surface, the interchange mechanism driven by gradient \( B \) and curvature drifts [Richmond, 1973] and by the centrifugal force [Lemaire, 1974] work together in the same direction, and transport small-scale plasma elements outward.

Such an interchange motion of small-scale field aligned inhomogeneities will result in rather inhomogeneous and fluctuating ionospheric motions and electric fields like those observed by Mozer [1973]. We consider that interchange of flux tubes in the nightside plasmadough region can be responsible for peeling off the plasmasphere beyond the \( \text{Roche Limit} \) surface.

References


