Third Cycle in Astronomy: THE SOLAR WIND (2 December 1975) - J. LEMAIRE

7.- Effects of rotation, viscosity, conductivity, heating, magnetic forces and waves on the solar wind expansion

Hydrodynamic two fluid models \( (T_e \neq T_p) \)

1) Conductive (Hartle and Sturrock, 1967)

hypothesis: steady state

- spherical symmetry
- radial expansion
- pressure tensor: isotropic
- no, or, radial magnetic field
- no viscosity
- no rotation

heat conduction: \( q_e = -8 \times 10^{-7} T_e^{5/2} \frac{dT_e}{dr} \)

\[
q_p = -3 \times 10^{-8} T_p^{5/2} \frac{dT_p}{dr}
\]

equations of transport:

\[
\frac{d}{dr} (\rho v r^2) = 0
\]

\[
\rho v \frac{dv}{dr} + \frac{d\rho}{dr} = -\rho \frac{C/\rho}{r^2}
\]

\[
\frac{3}{2} \rho_p \frac{d}{dr} \ln \rho_p \rho_p^{-5/3} = \frac{1}{2} \frac{d}{dr} (r^2 q_p) + \frac{3}{2} v n k (T_e - T_p)
\]

\[
\frac{3}{2} \rho_e \frac{d}{dr} \ln \rho_e \rho_e^{-5/3} = \frac{1}{2} \frac{d}{dr} (r^2 q_e) - \frac{3}{2} v n k (T_e - T_p)
\]

\[
p = p_e + p_i = n k (T_e + T_p)
\]

\[
\rho = n (m_p + m_e)
\]

\[
n = n_p = n_e
\]

\[
v = 9 \times 10^{-9} n T_e^{-3/2}
\]

energy exchange collision frequency
Results

- good agreement of $n_e(r)$ with observations between $2 R_\odot$ and $20 R_\odot$ (Fig. 1).
- discrepancy for $r < 2 R_\odot$ probably due to energy dissipation in this region.
- the decoupling of the electron and proton gases is illustrated in Fig. 2 ($T_e > T_p$).
- $T_e \propto r^{-2/7}$ as in Chapman's model where the heat conduction is the dominant term in eq. of energy.
- $T_p \propto r^{-6/7}$; since $\nu_p \ll \nu_e$ the protons expand adiabatically at large distance.
- Table 1 shows that the density at 1 AU is too large 100%, bulk speed too small (20%) ; electron temperature (200%) too large ; proton temperature (10 times) too small.

2) Conductive models + energy source beyond $2 R_\odot$

- Hartle and Barnes (1970) quote that heat must be deposited beyond $2 R_\odot$ by fast magnetosonic Alfvén waves. They propose a 2 fluid model with an empirical heat source function to increase the proton temperature at 1 AU up to the observed value (See Fig. 3, Table 1)
- Barnes, Hartle and Bredekkamp (1971) describe a solar wind model with dissipation beyond $2 R_\odot$ of 5.2 min. period Alfvén waves (See results Table 1)
- Increasing fluxes of Alfvén waves ($F_w > 2 \times 10^{26}$ erg/sec) lead to higher proton temperatures and bulk speeds at 1 AU. In agreement with Burlaga and Ogilivie (1970 observations).
3) Conductive, viscous models with magnetically reduced transport coefficients

- As a consequence of rotation ($\omega \neq 0$) streamlines and magnetic field lines are spirals.
- The observed expansion is not purely radial ($v_\phi = 8$ km/sec at 1 AU)
- $B_\phi$ and $v_\phi$ are related by

$$m r v_\phi - \frac{r B_r B_\phi}{4 \pi n v r} = L$$

$$v_\phi = r \left( \frac{v^2}{c_A^2 m \omega r^2} - 1 \right)$$

(from azimuthal component of equation of motion)

$$c_A^2 = \frac{B_r}{4\pi n m}$$

- Alfvénic critical point at $r_A$, where $v_r = c_A$ and $L = m \omega r_A^2$
- The non-radial component of the solar wind velocity predicted by model calculations is generally smaller than the observed 8 km/sec.
- Flux of angular moment carried by plasma and magnetic field

$$n v_r m v_\phi r 1 \text{AU} + \frac{B_r B_\phi}{4\pi r} r 1 \text{AU} = (4.8 \times 10^3 + 1.5 \times 10^3) \text{gm sec}^{-2}$$

This brakes the solar rotation in $6 \times 10^9$ years.
- Reduction of radial heat flux and tensions by non-radial magnetic fields

$$q_r = -\kappa \frac{dT}{dr} \cos^2 \phi$$

$$\tau = \frac{4}{3} \eta r \frac{d}{dr} \left( \frac{v}{r} \right) \cos^2 \phi$$

$\phi$ angle between $\vec{B}$ and $\vec{r}$
\[ \cos^2 \phi = \frac{v_r^2}{v_r^2 - (v_\phi - w r \sin \theta)^2} \]

(\( \theta = \) colatitude)

equations of transport :
\[ \rho v r^2 = \text{const.} \]

\[
\rho v \frac{dv}{dr} + \frac{dp}{dr} = -\rho \frac{G \mu}{r^2} + \frac{1}{c} (j \times B) + \frac{1}{r^3} \frac{d}{dr} r^3 \left( \tau_p + \tau_e \right)
\]

\[
\frac{3}{2} v_p \frac{d \ln \rho}{dr} = \frac{1}{r^2} \frac{d}{dr} \left( r^2 q_p \right) + \frac{3}{2} v n k (T_e - T_p) + \frac{1}{r^2} \frac{d}{dr} \left( r^2 \tau_p \right)
\]

\[
\frac{3}{2} v_e \frac{d \ln \rho}{dr} = \frac{1}{r^2} \frac{d}{dr} \left( r^2 q_e \right) - \frac{3}{2} v n k (T_e - T_p) + \frac{1}{r^2} \frac{d}{dr} \left( r^2 \tau_e \right)
\]

Results:

- As a consequence of viscous heat deposition the bulk speed at 1 AU is reduced (see 7th line in Table 1) Wolf Brandt and Southwick (1971) (see also figure 3)
- To obtain better results at 1 AU Wolf et al. (1971) reduce arbitrarily the transport coefficients between the base of the corona at 1 AU. The possible existence of wave particle interactions is quoted as a justification for such an artificial reduction of \( \nu \) and \( \eta \) (see 8th line in Table 1).
Unfortunately no satisfactory theory of transport coefficients for particle - waves - particle interactions is available presently.
Conclusion:

The most elaborated hydrodynamic models presently available cannot match both the observations in the inner corona and at 1 AU very satisfactorily without introducing some ad hoc assumption as for instance:

1) artificial reduction of transport coefficients (due to wave-particle interactions) or

2) addition of arbitrary heat source beyond 2 R☉

Therefore modelling the solar wind is still an open question.
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<th>1 A.U.</th>
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<tr>
<td></td>
<td>$R_\odot$</td>
<td>$10^8$ cm$^{-3}$</td>
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<td>Observations (average, quiet)</td>
<td>$r_o$</td>
<td>$n_o$</td>
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<tr>
<td>1 fluid conductive</td>
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<td>1 fluid conductive viscous</td>
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Pottasch (1960), Hundhausen (1972), Whang and Chang (1965), Hartle and Sturrock (1967), Hartle and Bernes (1970), Barnes et al. (1971), Whang et al. (1965), Wolf et al. (1971), Whang (1972)


CHAPMAN, S., Notes on the solar corona and the terrestrial ionosphere,


The density $n(r)$ predicted by a two-fluid model of the coronal expansion [3.22], along with coronal densities observed by Blackwell 1956 (as open circles) and by Michard 1954 (as crosses).

The temperatures $T_e(r)$ and $T_p(r)$ predicted by the two-fluid model (solid lines) and the single temperature $T(r)$ predicted by the one-fluid model (dashed line). The same coronal temperature $T(r_0) = 2 \times 10^6$ K has been assumed in both models.
Fig. 3  The temperatures $T_e(r)$ and $T_p(r)$ predicted by the basic two-fluid model (dashed lines) and the $T_e(r)$ predicted by two-fluid model incorporating an extended energy source (solid line).

Fig. 4  The electron and proton temperatures $T_e(r)$ and $T_p(r)$ predicted by the two-fluid models of Hartle and Barnes [77] (dashed lines) and Wolff, Brandt, and Southwick [71] (solid lines).