DIFFERENCES BETWEEN SOLAR WIND PLASMOIDS AND IDEAL MAGNETOHYDRODYNAMIC FILAMENTS

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(Received 26 March 1981)

Abstract—Plasma irregularities present in the solar wind are plasmoids, i.e. plasma-magnetic field entities. These actual plasmoids differ from ideal magnetohydrodynamic (MHD) filaments. Indeed, (1) their "skin" is not infinitely thin but has a physical thickness which is determined by the gyromotion of the thermal ions and electrons, (2) they are of finite extent and their magnetic flux is interconnected with the interplanetary magnetic flux, (3) when they penetrate into the magnetosphere their magnetic field lines become rooted in the ionosphere (i.e. in a medium with finite transverse conductivity), (4) the external Lorentz force acting on their boundary surface depends on the orientation of their magnetic moment with respect to the external magnetic field, (5) when their mechanical equilibrium is disturbed, hydromagnetic oscillations can be generated. It is also suggested that the front side of all solar wind plasmoids which have penetrated into the magnetosphere is the inner edge of the magnetospheric boundary layer while the magnetopause is considered to be the surface where the magnetospheric plasma ceases to have a trapped pitch angle distribution.

1. INTRODUCTION

It was suggested in 1976, at the European Geophysical Society symposium on the "Magnetopause regions", that the solar wind plasma carries small scale filamentary density irregularities which penetrate impulsively into the magnetosphere (Lemaire and Roth, 1978). These irregularities are plasmoids (i.e. plasma-magnetic field entities) according to the definition given by Bostik (1956). Contrary to steady-state interaction models, this time-dependent penetration mechanism explains a wider range of magnetospheric events which are typically impulsive phenomena (Lemaire, 1977). Lemaire et al. (1979) suggested also that the direction of the interplanetary magnetic field controls the impulsive penetration of small scale magnetosheath plasma irregularities into the magnetosphere.

In this paper we elaborate further on this time-dependent impulsive penetration model, and emphasize in which respects it differs from ideal magnetohydrodynamic (MHD) models. It is indicated in Section 2 that plasmoids of finite extent produce a magnetic field distribution which can interconnect with any external magnetic field, even though such a magnetic coupling does not exist in the case of infinitely conducting MHD filaments. The total Lorentz force acting on the surface of a plasma element has been decomposed into an internal force and an external force (Section 3). The external Lorentz force, as well as the "entry condition", depends on the orientation of the magnetic moment of the plasmoid or of the current layer with respect to the external magnetic field. The mechanical equilibrium condition for a plasmoid is discussed in Section 4. In the absence of mechanical equilibrium the front edge of a plasma element intrudes into the region of closed geomagnetic field lines. The inner edge of the magnetospheric "plasma boundary layer" is then defined in Section 5 as the envelope of all these plasma fronts. The outer edge of this transition region, which corresponds also to the "magnetopause", is defined in terms of characteristic plasma properties instead of magnetic field signatures.

2. MHD AND NON-MHD FILAMENT MODELS

The plasma irregularities considered by Lemaire and Roth (1978) have dimensions which are smaller than the diameter of the magnetosphere. Furthermore, the finite volume does not necessarily coincide with a magnetic flux tube; their whole surface is not a tangential discontinuity as it is for the ideal MHD models recently studied by Schindler (1979).

Figure 1(a) represents a finite plasma element with a sketch of the magnetic field line topology associated with such a density irregularity. By contrast, Fig. 1(b) shows an ideal MHD filament confined within a cylindrical magnetic flux tube of
The diamagnetic currents ($J_d$) circulating at the edge of the plasma element produce a magnetic field inside as well as outside its volume, which has dipole as well as multipole components. The current sheet is at least a few ion gyro-radii thick. The magnetic polarization $M$ of the plasmoid is related to the total current density ($K = \int J_d \, dN = M \times N$). The magnetic field has a non-zero component in the direction of $N$, the normal to this surface; this surface is not everywhere a tangential discontinuity. Thermoelectric charge separation produce electrostatic potential differences across the surface of the plasmoid. These polarization electric fields have components perpendicular and parallel to the magnetic field direction. When the external kinetic and magnetic pressures are too small, the volume of the plasma element expands with a velocity $v$ parallel to $N$.

In infinite length. In this MHD filament, plasma particles drift along (and "with") magnetic field lines which are equipotential lines as a consequence of the following assumed MHD conditions: (1) infinite parallel conductivity ($\sigma_1 = \infty$), with the consequence that $E \cdot B = 0$, (2) vanishingly small transverse Pedersen conductivity everywhere along the magnetic field lines ($\sigma_1 = 0$). Strictly speaking these MHD conditions are almost never satisfied except in collisionless magnetized plasmas that have either infinite extent, or are surrounded by insulating walls.

In ideal MHD models the lateral plasma boundaries are tangential discontinuities. Indeed, if this were not so, field lines would dip across the surface of the plasma cloud (as, for instance, in the case of classical double layers, oblique electrostatic shocks or rotational discontinuities). Along the magnetic field lines the high-speed plasma electrons would be able to run away out of the plasma element, unless there was a parallel electric potential barrier preventing these electrons from diffusing outward faster than the thermal ions. Obviously, a parallel potential difference implies a parallel charge separation electric field, and non-equipotential magnetic field lines; but since this would violate one of the MHD conditions ($E \cdot B = 0$), it is concluded that in the ideal MHD models the magnetic field lines must indeed be parallel to the plasma boundaries.

On the contrary, for irregularities like that illustrated in Fig. 1(a), the magnetic field lines are not equipotential lines, nor are they everywhere parallel to the plasma surface. Consequently, the filament model of Lemaire et al. (1979) is fundamentally different from the ideal MHD models.

Furthermore, in ideal MHD models, it is assumed that the magnetic field vector changes abruptly direction and/or intensity from a constant value ($B_F$) inside the filament, to some different value ($B_M$) outside the filament. However, actual tangential discontinuities always have a finite thickness. A more realistic picture is then one where the magnetic field changes gradually over several ion gyro-radii as described in papers by Lemaire and Burlaga (1976), Roth (1976, 1978, 1979, 1980) and more recently by Lee and Kan (1979, 1980). In other words, the "skin" of the plasma cloud is not infinitely thin but has a physical thickness, in the direction perpendicular to $B$, which is determined by the gyromotion of the thermal ions and electrons.
The difference between ideal MHD filaments and non-MHD plasma clouds is best illustrated by the comparison between, on the one hand, "an infinitely long and infinitely conducting solenoid (or magnetized rod)" and, on the other hand, "a solenoid (or a magnetic bar) of finite length". Indeed, like the solenoid of finite length, a diamagnetic plasma cloud produces a dipole-like magnetic field outside its surface (see Fig. la). The magnetic flux through any cross-section of the plasma element is coupled to (i.e. interconnected) and equal in magnitude to the total flux outside the plasma cloud. The magnetic field vector has, in general, a component normal to the surface of the plasma irregularity; magnetic field lines run across the plasma boundaries. However, when this plasma element is stretched out infinitely in the magnetic field direction, the field intensity tends to zero outside the element, as for an infinitely long solenoid. Furthermore, the field inside infinitely long MHD filaments or solenoid is uniform; the magnetic field lines are everywhere parallel to their surface. As for an infinitely long superconducting solenoid brought into an external field, there is no magnetic connection (or coupling) between the inside and the outside of the ideal MHD filament. When the infinitely long and superconducting solenoid is introduced into an external magnetic field, the total magnetic energy of the system does not depend on its orientation with respect to the external field. This is not the case for a solenoid of finite length which can be either accelerated or decelerated in an external dipole magnetic field, depending on the relative orientation of their respective magnetic moments. This classical example indicates that modelling 3-D systems in terms of idealized 2-D or 1-D solutions is not always appropriate.

Finally, it would not be realistic to consider that the Pedersen conductivity ($\sigma_\parallel$) is nearly zero everywhere along geomagnetic field lines. Indeed, magnetic field lines are usually rooted in the ionosphere (i.e. in a medium with finite transverse conductivity), and hence the second MHD condition is not satisfied:

$$\Sigma_p = \int_\Omega \sigma_\parallel \, dh = 0.$$  

Because of these fundamental differences, it seems difficult to draw valid conclusions about the motion of non-MHD and finite plasma filaments in an external magnetic field, from theoretical calculations based on ideal MHD models of infinitely long filaments. Similarly, it is not obvious that a stationary solution (usually proposed for convenience because of its mathematical simplicity) is appropriate to describe a physical mechanism which is inherently based on time-dependent processes.

Fälthammar et al. (1978) describe many other situations in geophysical and astrophysical plasmas where such remarks can directly be applied.

### 3. Lorentz Force on Plasmoids

Let us first consider an unmagnetized cloud of plasma in a vacuum devoid of any external magnetic field. The plasma pressure inside the cloud is unbalanced since there is neither kinetic nor magnetic pressure outside the filament, hence the pressure gradient force pushes the plasma front outwards, the volume occupied by the plasma expands indefinitely and its density tends to zero. An electric potential drop extending over a distance of a few Debye lengths across the moving boundary maintains local and global quasi-neutrality in the whole plasma blob. The height of this potential barrier is a few times $kT/e$, where $T$ is the electron temperature. This electric potential is produced by thermoelectric charge separation at the front of the plasma density element.

Cosmic plasma clouds are usually magnetized. The magnetic flux through a cross-section of a plasma irregularity depends on the intensity of the magnetic field in the source region in which the plasma has been formed by ionization of the original neutral gases.

The magnetic field distribution ($B$) inside and in the vicinity of the ionized cloud is generated by magnetization currents, gradient $B$ and curvature currents ($J_d$) circulating at the surface of the collisionless plasma element as illustrated in Fig. la.

Such a plasma-magnetic field entity has been called "plasmoid" by Bostik (1956).

The net force ($F$) acting on the matter contained in a volume element ($V$) is the sum of the "pressure gradient force, the Lorentz force, and the electric force".

$$F = - \int_V \text{div} \, P \, dV + \int_V (J_d \times B) \, dV + \int_V qE \, dV,$$

where $P$ is the kinetic pressure tensor for the electrons and ions; $E$ is the electric field and $q$ is the electric charge density satisfying Poisson's
equation

\[ \text{div } \mathbf{E} = q. \]  \hspace{1cm} (2)

From this, and from Maxwell's equation

\[ \mathbf{J}_d = \frac{1}{\mu_0} \text{curl } \mathbf{B}, \]  \hspace{1cm} (3)
eqn. (1) becomes

\[ \mathbf{F} = -\int_V \left[ \mathbf{P} + \left( \frac{\mathbf{B}^2}{2\mu_0} + \frac{\varepsilon_0}{2} \mathbf{E}^2 \right) \mathbf{1} - \varepsilon \mathbf{E} \right] \mathbf{E} \] 
\[ - \frac{1}{\mu_0} \mathbf{B} \mathbf{B} \right] \, dV. \]  \hspace{1cm} (4)

Charge separation electric fields \( \mathbf{E} \) can be large at the edges of the plasmoid. They determine the distributions of the electrons and ions separately near the plasma surface. But, since the net charge density \( q \) is extremely small, the electrostatic force on the whole plasma (electrons + ions) contributes little to \( \mathbf{F} \). In other terms

\[ \varepsilon_0 \mathbf{E}^2 \ll \frac{\mathbf{B}^2}{\mu_0}, \]

or, what is equivalent, the electric drift velocity \( \mathbf{v} \) is much smaller than the speed of light in a vacuum \( c \):

\[ \frac{\mathbf{E}}{\mathbf{B}} = \mathbf{v} \ll (\mu_0 \varepsilon_0)^{-1} = c. \]

Using Stokes' theorem, eqn. (4) can then be written as a surface integral:

\[ \mathbf{F} = -\int_S \mathbf{N} \cdot \left( \mathbf{P} + \frac{\mathbf{B}^2}{2\mu_0} - \frac{1}{\mu_0} \mathbf{B} \mathbf{B} \right) \, dS, \]  \hspace{1cm} (5)

where \( \mathbf{N} \) is the unit vector normal to the surface \( (S) \) of the plasmoid. This equation has been used by Schindler (1979) to show that there is a force acting on a curved MHD filament which tends to reduce the curvature of the magnetic field lines and to straighten them.

Equation (5) can, for instance, be used to evaluate the net force acting on the surface of a planar directional discontinuity. If \( \mathbf{B}_0 \) and \( \mathbf{B}_1 \) are the magnetic field intensities on both sides of the discontinuity, it can be shown that the force acting on a cylindrical mass element, with a unit cross-section in the vicinity of this surface, is given by:

\[ \mathbf{F} = \mathbf{N} \cdot (\mathbf{P}_1 - \mathbf{P}_0) + \frac{1}{2\mu_0} (\mathbf{B}_1^2 - \mathbf{B}_0^2) \mathbf{N} \]
\[ - \frac{1}{\mu_0} \mathbf{N} \cdot \mathbf{B}_0 (\mathbf{B}_1 - \mathbf{B}_0). \]  \hspace{1cm} (6)

To obtain this equation, we have taken into account the fact that the normal component of \( \mathbf{B} \) is continuous across the surface of discontinuity, i.e. \( \mathbf{N} \cdot \mathbf{B}_0 = \mathbf{N} \cdot \mathbf{B}_1 \). If \( P^N \) denotes the component of pressure component parallel to \( \mathbf{N} \), the normal component of \( \mathbf{F} \) is equal to

\[ \mathbf{N} \cdot \mathbf{F} = P^N_1 - P^N_0 + \frac{1}{2\mu_0} (\mathbf{B}_1^2 - \mathbf{B}_0^2). \]  \hspace{1cm} (7)

The magnetic effect of the surface current with a density

\[ \mathbf{K} = \int_\Delta \mathbf{J}_d \, dN \]  \hspace{1cm} (8)
is equivalent to that of a magnetized medium whose magnetic polarization is zero outside the plasmoid, and equal to \( \mathbf{M} \) inside the plasma element such that

\[ \mathbf{K} = \mathbf{M} \times \mathbf{N}. \]  \hspace{1cm} (9)

Integrating the three components of eqn. (3) along the normal direction \( \mathbf{N} \), one obtains

\[ \mathbf{K} = \frac{1}{\mu_0} (\mathbf{B}_1 - \mathbf{B}_0) \times \mathbf{N}. \]  \hspace{1cm} (10)

Therefore, we define the magnetic polarization inside the plasmoid as

\[ \mathbf{M} = \frac{1}{\mu_0} (\mathbf{B}_1 - \mathbf{B}_0). \]  \hspace{1cm} (11)

Since the normal components of \( \mathbf{B}_1 \) and \( \mathbf{B}_0 \) are equal, it results that \( \mathbf{M} \) is parallel to the surface of discontinuity. Equation (7) can also be written as

\[ \mathbf{N} \cdot \mathbf{F} = (P^N_1 - P^N_0) + \frac{\mathbf{B}_0}{2} M^2 + \mathbf{M} \cdot \mathbf{B}_0. \]  \hspace{1cm} (12)

The first term in eqn. (12) corresponds to the excess of perpendicular kinetic pressure inside the plasmoid, while the second term is the internal Lorentz force produced by the plasma currents.
Differences between solar wind plasmoids and ideal magnetohydrodynamic filaments

Like surface tension acting on a droplet of water, this internal electromagnetic force maintains the cohesion of the plasma filament as an entity. This force normal to the surface of the plasma irregularity does not depend on the direction of the vector $\mathbf{M}$.

The third term of eqn. (12) corresponds to the normal component of the "external Lorentz force":

$$\int_V \mathbf{J}_d \times \mathbf{B}_0 \, dV = N(\mathbf{M} \cdot \mathbf{B}_0) - M(N \cdot \mathbf{B}_0).$$  \hspace{1cm} (14)

Note that eqn. (14) is identical to eqn. (5) of Lemaire et al. (1979). This external Lorentz force results from the interaction between the external magnetic field $\mathbf{B}_0$ and the local plasma currents $\mathbf{J}_d$, while the normal component of this force (i.e. $\mathbf{M} \cdot \mathbf{B}_0$) corresponds to the third term in eqn. (12). Note that the last terms in eqns. (14) and (6) are identical and that these terms, $\mathbf{M} (N \cdot \mathbf{B}_0)$, differ from zero only when $\mathbf{B}_0$ has a non-zero component perpendicular to the plasma boundary; for instance, in the case of a rotational discontinuity, when $N \cdot B_0 \neq 0$. For tangential discontinuities,

$$N \cdot B_0 = N \cdot B_i = 0,$$  \hspace{1cm} (15)

and hence the last terms are zero.

In the case of finite plasmoids the internal and external Lorentz forces contain additional multipolar terms depending upon the magnetic field gradients parallel to the surface of the plasma elements. The contribution of these terms will be studied in a forthcoming paper.

From eqn. (12), it can be concluded that the normal component of the total Lorentz force is the sum of an internal force (13) independent of the direction of $\mathbf{M}$, and an external force (14) which depends on the angle between the vector $\mathbf{M}$ and the external magnetic field $\mathbf{B}_0$. Consequently, for fixed values of $|\mathbf{M}|$ and $|\mathbf{B}_0|$, the normal components of the external Lorentz force and of the total Lorentz force are maximum when $\mathbf{M}$ and $\mathbf{B}_0$ are parallel. Note, however, that the normal component of the total Lorentz force does not change when the magnetic field inside the plasma filament is changed from $+\mathbf{B}_i$ to $-\mathbf{B}_i$. This result can easily be seen from eqn. (7), and it has been used by Schindler (1979) to show that the motion of an ideal MHD filament is the same whether the fields inside and outside are parallel or antiparallel. This may appear to contradict the previous conclusion, but this is not so, because changing $\mathbf{B}_i$ into $-\mathbf{B}_i$ is not equivalent to changing $\mathbf{M}$ into $-\mathbf{M}$.

4. CONDITION FOR MECHANICAL EQUILIBRIUM

When the condition

$$P_i^N - P_o^N + \frac{1}{2\mu_o} (B_i^2 - B_o^2) = 0$$  \hspace{1cm} (16)

is satisfied or, what is equivalent, when

$$P_i^N - P_o^N + \frac{1}{2\mu_o} M^2 + M \cdot B_0 = 0,$$  \hspace{1cm} (17)

it can be seen, from eqn. (7) or eqn. (12), that the normal component of $\mathbf{F}$ is equal to zero as is also the acceleration of the plasma boundary. The plasma element is then in mechanical equilibrium. These equilibrium conditions (16) or (17) can never be satisfied when the external magnetic field $\mathbf{B}_0$ and the kinetic pressure $P_o^N$ at large radial distance outside the plasmoid are both equal to zero, as in a vacuum. In this case, the magnetized plasma cloud will expand adiabatically; the larger its magnetic pressure, the larger its expansion rate. In general, however, the plasmoid is not in a vacuum and it will expand until the kinetic and magnetic pressures inside the plasma cloud reach the values required to satisfy the equilibrium conditions (16) or (17), i.e. until the total pressures on both sides of the plasma surface are balanced. During this process, the cloud boundary will move outwards and its volume will increase, while its density, magnetic pressure and perpendicular kinetic pressure will all decrease so as to reduce the total pressure unbalance. It can easily be envisaged that overshooting due to the inertia of the mass element can lead to over-expansion and to subsequent contraction and expansion phases of the plasma element. As a result, hydromagnetic oscillations can be generated and propagate in the external plasma as Alfvén waves; continuous geomagnetic pulsations often occurring as a train of separate $Pc$ 3–4 wave packets like those recently analysed by Mier-Jedrzejowicz and Hughes (1980), can be initiated by impulsive penetrations of separate solar wind plasmoids in the dayside magnetosphere.
5. A TIME-DEPENDENT AND IRREGULAR MAGNETOPAUSE SURFACE

The plasma and field on both sides of a steady-state magnetopause in mechanical equilibrium satisfy the equilibrium conditions (16) or (17) until the instant when the magnetosheath plasma pressure and momentum density are suddenly enhanced by the arrival of some new plasma cloud reaching the magnetopause region. Only solar wind plasma irregularities with an excess momentum are able to make their way through the magnetosheath and to reach the average magnetopause position (Lemaire and Roth, 1978). Because of the excess momentum and also because of the additional plasma pressure, the surface separating magnetosheath plasma and magnetospheric plasma moves towards the Earth. Magnetospheric plasma can also be pushed ahead towards the Earth by the magnetosheath plasma front acting as a piston. But small-scale solar wind irregularities intruding into the magnetosphere become permeated by closed geomagnetic field lines, and trapped particles of magnetospheric origin drift into these plasma clouds.

The admixture of magnetosheath-like and magnetospheric-like particle populations is typical of the “boundary layer plasma” found at the outermost fringes of the magnetosphere (Eastman and Hones, 1979). The front side of this intruding plasma element is part of the inner edge of the “plasma boundary layer”, while the outer edge of the so-called “plasma boundary layer” is the magnetopause per se. Every time an intruding plasma element disturbs the local geomagnetic field distribution, the field lines which were originally closed eventually become interconnected with those of the solar wind. Similarly, the inner edge of the “plasma boundary layer” is formed by the front sides of all the solar wind plasma irregularities that have sufficient momentum to penetrate into the closed magnetic field lines region. This excess of momentum is transferred to the dayside cusp ionosphere as described by Lemaire (1979). The excess kinetic energy of the intruding plasma is dissipated by Joule heating of the ionospheric plasma in the cleft regions (Lemaire, 1977) where Titheridge (1976) has detected a large temperature peak from 400 up to 1000 km altitude. Every time a new plasma density irregularity reaches the average magnetopause position and increases the plasma momentum and pressure in the magnetosheath, new solar wind plasma breaks into the geomagnetic field like an ocean wave breaking on a sandy beach. The plasma boundary layer can be compared to the thin transition layer where air bubbles are engulfed below the surface of an ocean disturbed by a gusty air flow (Lemaire, 1978).

In the impulsive penetration model, the magnetopause is not a smooth stationary surface, but a rather irregular and time-varying boundary between plasma of different origins. Furthermore, in disagreement with conventional interaction models, not all the solar wind particles impinge on a stationary and sharply defined magnetopause surface; indeed, plasma elements traversing the bow shock with a momentum density smaller than the average will not reach the magnetopause region, but will be deflected sideways at a greater distance from the Earth and will slip around the flanks of the magnetosphere in the outermost layers of the magnetosheath.

Finally, in the impulsive penetration model, the magnetopause is defined as the surface where the magnetospheric plasma ceases to have a trapped pitch angle distribution. Thus this surface separates the closed geomagnetic field lines from those that have at least one “foot” in the interplanetary medium. This definition of the magnetopause is based on the difference between the plasma properties found on the two sides, and not on the observed magnetic field signatures. Indeed, the type of magnetic field lines can change from “closed” to “open” (interconnected) without a sharp variation in either the intensity or the direction of the magnetic field.

REFERENCES


Differences between solar wind plasmoids and ideal magnetohydrodynamic filaments


