Infrared cooling by atomic oxygen in the thermosphere

by G. KOCKARTS
INTRODUCTION

In a study of the upper atmosphere thermal balance, a knowledge of the heat sources and the heat sinks is of fundamental importance. Numerous satellite drag analyses have confirmed that solar ultraviolet radiation is the principal physical agent for heating of the thermosphere heating. Above the mesopause (∼85 km), the hydrodynamical regime is such that heat is mainly transported by conduction (Nicolet, 1961). The following hypothesis are adopted as a first approximation:

(1) gravity is the only external force acting on the atmosphere,

(2) diffusive equilibrium exists among the atmospheric constituents,

(3) the mean mass transport velocity is neglected, the heat conduction equation can then be written (Chapman and Cowling, 1952).

\[ \rho c_v \frac{\partial T}{\partial t} + \text{div } \dot{Q} = P - L \]  

(1)

where \( \rho \) is the total density, \( c_v \) is the specific heat at constant volume, \( T \) is the absolute temperature, \( t \) is the time, \( \dot{Q} \) is the conduction heat flow, \( P \) is the heat production and \( L \) is the heat loss. Nicolet (1961) has clearly shown the effect of heat production \( P \) due to the solar ultraviolet radiation which is mainly absorbed between 100 km and 200 km by atmospheric components such as \( O, O_2 \) and \( N_2 \). An important factor which is however not known is the heating efficiency of the ultraviolet radiation as a function of wavelength and atmospheric components. In practice the amount of solar energy absorbed in the thermosphere cannot be completely converted into heat. Additional processes, such as atomic and
molecular ionization, vibrational and rotational excitation of the molecular species, should be taken into account. Recently, Izakov and Morozov (1970) have made an estimate of the heating efficiency and have emphasized the effect of the heat input in the Schumann-Runge continuum below 1750 Å.

The loss process compatible with equation(1), is an energy degradation through infrared emission. Atoms or molecules emitting in the infrared must have a permanent magnetic dipole; the transition probability can then be high enough to make the emission important for a heat loss. Among the atmospheric constituents, i.e. O₂, N₂, O, He and H, which can play a role in the total concentration of the upper atmosphere, only atomic oxygen has a permanent magnetic dipole. Nitric oxide, introduced by Nicolet (1945) to explain the D region ionization, can also emit in the infrared. The NO concentration however is too small in the thermosphere to affect the heat budget, therefore the infrared cooling by atomic oxygen emission only will be considered in this study. The importance of this effect was first pointed out by Bates (1951) and recently, Feldman and McNutt (1969) and Houck and Harwit (1969) were able to detect the 63 μm emission of atomic oxygen.

2. EXCITATION AND DE-EXCITATION PROCESSES

The ground state of neutral atomic oxygen is a triplet (see fig. 1) for which the \(^3\text{P}_0\) and \(^3\text{P}_1\) levels lie only 226.5 cm\(^{-1}\) and 158.5 cm\(^{-1}\) above the ground level \(^3\text{P}_2\). The excitation potential involved, i.e. 0.028 eV and 0.020 eV are of the same order of magnitude as the thermal energy of the atmospheric constituents. Since collisions are frequent in the thermosphere, Bates (1951) suggested the following excitation mechanism

\[
X + O(^3\text{P}_J) + X + O(^3\text{P}_J) \quad (2)
\]
Fig. 1.- Energy level diagram of atomic oxygen ground state
where $X$ is any atmospheric constituent, and the quantum number $J$ is equal to 2 or 1 and $J'$ is equal to 1 or 0. If process (2) is fast enough, it is reasonable to assume that the oxygen atoms are distributed at the levels 0, 1 and 2 according to a Boltzmann distribution. The concentration $n(0, \frac{3}{2}P_J)$ is therefore given by

$$n(0, \frac{3}{2}P_J) = n(0) \sum_{J=0}^{2} g_J \exp\left(-\frac{E_J}{kT}\right)$$

where $E_J$ is the excitation energy for the level $J$, $g_J = 2J + 1$ is the statistical weight, $k$ is the Boltzmann constant and $n(0)$ is the total atomic oxygen concentration. For temperatures ranging from $200^\circ K$ to $2000^\circ K$, the relative abundance of the $\frac{3}{2}P_1$ state varies between 0.16 and 0.31. The infrared energy emitted by unit volume and in unit time in a transition $J' \to J$ is then given by

$$L = E_{J'} A_{J'-J} n(0, \frac{3}{2}P_J)$$

where $A_{J'-J}$ is the spontaneous transition probability. Using the numerical values given in fig. 1 and the relation (3) one immediately obtains

$$L_{63\mu} = \frac{1.69 \times 10^{-18} n(0) e^{-228/T}}{1 + 0.6 e^{-228/T} + 0.2 e^{-326/T}} \text{ erg cm}^{-3} \text{ sec}^{-1}$$

and

$$L_{147\mu} = \frac{4.59 \times 10^{-20} n(0) e^{-326/T}}{1 + 0.6 e^{-228/T} + 0.2 e^{-326/T}} \text{ erg cm}^{-3} \text{ sec}^{-1}$$

These relations show that, in a first approximation, the $147\mu$ emission is negligible compared to the $63\mu$ emission. The energy rate $R = L/n(0)$ emitted at $63\mu$ and at $147\mu$ is represented in fig. 2 as a function of temperature.
Fig. 2. - Emitted energy rate at 63 \( \mu \) and at 147 \( \mu \) as a function of temperature.
The relations (5a) and (5b) are valid, when the external radiation field is neglected. Before showing the effect of the external radiation field, it is necessary to define the specific intensity $I_v$ measured in erg cm$^{-2}$ sec$^{-1}$ ster$^{-1}$ Hz$^{-1}$ such that $I_v$ dv is the energy between the frequencies $v$ and $v + dv$ transported across a unit area perpendicular to the beam, in a unit solid angle per unit time. For the transitions between the two levels 1 and 2, where 1 corresponds to the upper level, the number of absorptions is proportional to $B_{21} I_v$ and the number of emissions is proportional to $A_{12} + B_{12} I_v$ (see fig. 3). The factors $B_{21}$ and $B_{12}$ are the Einstein coefficients of absorption and induced emission, respectively. The effect of induced emission is to reduce the effective absorption coefficient in the radiative transfer equation presented in section 4. The Einstein coefficients are related by

$$g_2 B_{21} = g_1 B_{12} \tag{6}$$

and

$$A_{12} = (2h \nu^3/c^2) B_{12} \tag{7}$$

where $g_1$ and $g_2$ are the statistical weights of the levels, $h$ is Planck's constant, $\nu$ is the frequency of the radiation and $c$ the speed of light.

The amount of absorbed energy is proportional to $n_2 B_{21} I_v$, and the emitted energy at frequency $v$ is proportional to $n_1 (B_{12} I_v + A_{12})$. The intensity of the radiation field depends therefore on the concentration $n_1$ and $n_2$ of the upper and lower level respectively. It has been assumed that $n_1$ and $n_2$ are given by the Boltzmann distribution (3), the subscripts corresponding to the quantum number $J$. This hypothesis is justified (Bates, 1951; Münch, 1962) in the terrestrial atmosphere, since the excitation process (2) takes place much faster than does spontaneous photon emission.

From the preceding discussion, it is clear that relations (5a) and (5b) represent the emitted energy only when absorption and induced emission are neglected. The importance of these processes is however directly related to the amount of atomic oxygen in the upper atmosphere.
Einstein coefficients

\[ B_{21} I_V \quad A_{12} + B_{12} I_V \]

Fig. 3.- Einstein relations
It is then intuitively evident that absorption of the $63\mu$ emission becomes important below a certain altitude. This altitude can only be determined by solving the radiative transfer equation discussed in section 4.

3. IMPORTANCE OF THE $63\mu$ EMISSION

It is possible to show (Nicolet, 1961) from equation (1) that the vertical heat flow $Q$ due to solar ultraviolet absorption is given by

$$Q = Q_{uv} (1 - e^{-\tau_{uv}})$$  \hspace{1cm} (8)

where $Q_{uv}$ is the solar ultraviolet flux at the top of the atmosphere and $\tau_{uv}$ is the optical depth resulting from absorption by all the atmospheric components. The maximum local heat production occurs approximately at the altitude where the optical depth $\tau_{uv}$ is unity. If $\tau_{uv} = 1$ at a height where $n(0) \approx 10^{10}$ cm$^{-3}$, i.e. around 150 km, the heat flow $Q$ is approximately equal to 1.2 erg cm$^{-2}$ sec$^{-1}$ for a solar flux of 2 erg cm$^{-2}$ sec$^{-1}$. At lower heights $Q$ is of course smaller.

When the $63\mu$ absorption is neglected, the vertical infrared flow $Q_{ir}(z)$ at altitude $z$ can be obtained by integrating (4),

$$Q_{ir}(z) = E_1 A_{12} \int_z^\infty n(0, 3P_1) dz$$  \hspace{1cm} (9a)

If the atmospheric temperature ranges from 300°K to 2000°K, the ratio $n(0, 3P)/n(0)$ varies between 0.2 and 0.3. The infrared cooling flow (9b), evaluated with the numerical data of fig. 1, is then given approximately in erg cm$^{-2}$ sec$^{-1}$ column$^{-1}$ by

$$Q_{ir}(z) \sim 7 \times 10^{-19} \int_z^\infty n(0) dz$$  \hspace{1cm} (9b)
In order to make a numerical estimate, it is necessary to adopt a vertical atomic oxygen distribution. Fig. 4 shows a possible atomic oxygen distribution briefly discussed by Kockarts and Peetermans (1970). The total contents of atomic oxygen above 120 km and 150 km are respectively $1.8 \times 10^{18} \text{ cm}^{-2}$ and $5.3 \times 10^{17} \text{ cm}^{-2}$ for a thermopause temperature of 750°K. These quantities show that the estimated infrared cooling flow (9b) is of the same order of magnitude or is even greater than the ultraviolet heating below 150 km. The preceding section indicates that expressions (9a) and (9b) overestimate the infrared loss below a certain height. In theoretical atmospheric models, the lower boundary conditions are usually fixed around 120 km or lower. Any solution of the heat conduction equation (1) should therefore take into account the infrared loss term. It is however necessary to know the degree of accuracy given by the approximations (4) or (9b).

4. RADIATIVE TRANSFER EQUATION

By counting the gains and losses of a pencil of radiation through a length of path $ds$ in the atmosphere, the radiative transfer equation can be written (Chandrasekhar, 1960)

$$\Delta \nu \frac{dI_\nu}{ds} = \left[ n_1 (A_{12} + B_{12} I_\nu) - n_2 B_{21} I_\nu \right] \left( \frac{hv}{4\pi} \right) \quad (10)$$

where $\Delta \nu$ is the total width of the 63$\mu$m line and $hv = E_1$ is the energy of the excited level giving rise to the 63$\mu$m emission. The introduction of $\Delta \nu$ has been made in order to avoid an integration over frequency. Actually this procedure is equivalent to assume that the emission line has a rectangular profile of width $\Delta \nu$. Equation (10) shows the terms due to spontaneous and induced emission and the absorption term.

If the direction $s$ makes an angle $\theta$ with the vertical $z$, measured positively from the Earth's surface ($dz = \cos ds = uds$), it is possible to transform equation (10) into
Fig. 4. Vertical atomic oxygen distribution for thermopause temperatures of 750 °K and 2000 °K.
where

\[
K_\nu = n_2 B_{21} h \nu \left[ 1 - \exp \left( -\frac{E_1}{kT} \right) \right] /4\pi \Delta \nu
\]

and

\[
\epsilon_\nu = A_{12} h \nu n_1 /4\pi \Delta \nu
\]

Relation (12) is obtained by using (3) and (6) to eliminate \( B_{12} \) and to express \( n_1 \) as a function of \( n_2 \). From (12), the optical depth derivative \( d\tau/dz \) is defined by

\[
d\tau/dz = - K_\nu
\]

With (14), equation (11) can finally be written

\[
udl /dz = - K_\nu I_\nu + \epsilon_\nu
\]

where

\[
B(T) = (2h\nu^3/c^2) \left[ \exp \left( \frac{E_1}{kT} \right) - 1 \right]^{-1}
\]

is the specific intensity at frequency \( \nu \) of a black body characterized by a temperature \( T \) depending on the altitude, i.e. on the optical depth.

It is reasonable in the Earth's upper atmosphere to adopt a Doppler profile for the 63\( \mu \) line. Fig. 5 gives the absorption cross section as a function of wavenumber expressed in Doppler widths. The cross section \( \sigma_0 \) at the center of the line is given by

\[
\sigma_0 = 9.4 \times 10^{-17} T^{-1/2} \text{ cm}^2
\]

and it can be obtained in the way described by Kockarts and Peetermans (1970).

As we are mainly interested in the total energy involved in the 63\( \mu \) line, the profiles shown in Fig. 5 may be approximated by rectangular profiles of width \( \Delta \nu \). The total Doppler width changes however with temperature, i.e. with height, according to
Fig. 5.- Atomic oxygen absorption cross section at 63 $\mu$ as a function of wave number expressed in Doppler width units.
\[ \Delta v = 2 \frac{v_0}{c} \left( \frac{2 kT \ln 2}{m} \right)^{1/2} = 8.51 \times 10^5 \ T^{1/2} \ (\text{Hz}) \] (18)

where \( v_0 \) is the frequency at the centre of the line and \( m \) the mass of an oxygen atom. This factor comes into the optical depth \( \tau \) and the result of the integration of (14) is presented in Fig. 6 (Kockarts and Peetermans, 1970).

When a value is adopted for the 63\( \mu \) line width, it is possible to approximate the frequency integrated intensity \( I = \int I_\nu \, d\nu \) by \( I_\nu \Delta v \). The solutions of the radiative transfer equation can therefore be multiplied by the total Doppler width to obtain a frequency integrated intensity in \( \text{erg cm}^{-2} \text{ sec}^{-1} \text{ ster}^{-1} \).

5. SOLUTION OF THE RADIATIVE TRANSFER EQUATION

The frequency integrated intensity \( I = I_\nu \Delta v \) can be obtained from equation (15) in the general form

\[ I = \Delta v \left( I_o \ e^{(\tau-\tau_o)/u} - u^{-1} e^{(\tau-\tau_o)/u} \int_{\tau_o}^{\tau} e^{(\tau_o - t)/u} B(t) \, dt \right) \] (19)

In (19), \( I_o \) is the specific intensity at the boundary level where the 63\( \mu \) optical depth is \( \tau_o \) and \( B(t) \) is the black body specific intensity which implicitly depends on \( \tau \), i.e. on temperature and height.

The intensity can be divided in a upward intensity \((u > 0)\) and a downward intensity \((u < 0)\). For each case, the boundary condition is different. For the descending radiation, Kockarts and Peetermans (1970) have shown that the specific intensity due to solar emission at 63\( \mu \) can be neglected at great altitudes where \( \tau_o = 0 \). In this case, the frequency integrated downward intensity is given from (19) by

\[ I_-(\tau,\mu) = -\Delta v \int_{\tau_o}^{\tau} e^{(\tau-t)/u} B(t) \frac{dt}{u} \] (20)
Fig. 6.- Vertical distribution of the optical depth at 63 μm for thermopause temperatures of 750 and 2000 °K.
For the ascending radiation, it is necessary to assume a black body emission \( B(T_E) \) at 63\( \mu \) coming from the vicinity of the tropopause. Such an emission can correspond to infrared water vapor rotational transitions. In the present study a tropopause temperature \( T_E = 190^\circ\text{K} \) is adopted. The choice of this value is not very critical for the infrared emission above 100 km, since above this height the 63\( \mu \) emission comes entirely from the ambient atomic oxygen. With these conditions, the frequency integrated upward intensity is given from (19) by

\[
I_+ (\tau, \mu) = \Delta \nu \int_\tau^{\tau_o} e^{(\tau-t)/u} B(T) \frac{dt}{u} + B(T_E) e^{(\tau-\tau_o)/u}
\] (21)

where \( \tau_o \) is the optical depth at the lower boundary. At 50 km altitude, \( \tau_o \) is of the order of 3.7.

The frequency integrated intensities can be experimentally determined and the recent observation by Feldman and McNutt (1969) has been discussed by Kockarts and Peetermans (1970) in relation to the radiative transfer process.

6. MEAN NET FLUX AND VOLUME EMISSION RATE

For the heat conduction equation, another quantity is however useful. It is the mean net flux \( F(\tau) \) resulting from an upward and a downward flux

\[
F(\tau) = F_+ (\tau) + F_- (\tau)
\] (22)

This flux is obtained by integrating \( I_+ \) and \( I_- \) over one hemisphere for \( u \) varying from 1 to 0 and from 0 to -1, respectively. In this way, the mean net flux is written

\[
F(\tau) = 2 \pi \Delta \nu \left[ \int_0^{\tau_o} B(t) E_2 (|\tau-t|) \, dt + B(T_E) E_3 (\tau_o - \tau) \right]
\] (23)
where the exponential integral $E_n(x)$ is defined by

$$E_n(x) = \int_0^1 e^{-x/u} u^{-2} du$$

(24)

and $|\tau - t|$ represents the absolute value of $\tau - t$. In the one dimensional heat conduction equation this mean net flux appears after a first integration over height, which leads to the heat flow transported by conduction. It has however been assumed that the atmosphere is plane parallel stratified and a three dimensional solution would require a more complex radiative transfer equation.

Fig. 7 shows the mean net flux computed for a thermopause temperature of $1000^\circ$K. Above $110$ km, $F$ is positive and the infrared emission is lost into interplanetary space. Below $90$ km, $F$ is positive or negative depending whether or not a lower boundary emission is introduced. The effect of the lower boundary emission is however negligible at altitudes above $100$ km. At $300$ km, the mean net flux, directed upwards, is equal to $0.1$ or $0.2$ erg cm$^{-2}$ sec$^{-1}$ for thermopause temperatures of $750^\circ$K and $2000^\circ$K respectively.

The volume emission rate $L$ (erg cm$^{-3}$ sec$^{-1}$), to be used in equation (1) is in fact given by the divergence of the mean net flux. $L(z)$ as a function of height is written as follows

$$L(z) = \frac{dF}{dt} \frac{dt}{dz}$$

(24)

By using the expressions (5a) and (23), equation (24) becomes

$$L(z) = L_{63\mu} [1 - X]$$

(25)

where

$$X = 0.5 \left[ e^{228/T} - 1 \right] \left\{ \int_0^{\tau^o} E_1(|\tau - \tau|) \left[ e^{228/T} - 1 \right]^{-1} d\tau \right.$$

$$+ E_2 (\tau_o - \tau) \left[ e^{228/T_E} - 1 \right]^{-1} \right\}$$

(26)
Fig. 3. Mean net flux for a thermopause temperature of 1000 °K. The dotted curve ($T_E = 0$) corresponds to the case where no lower boundary emission is present.
The term containing the second order exponential integral $E_2$ in (26) is due to the lower boundary emission. A numerical procedure that may be used to evaluate (26) has been described by Kockarts and Peetermans (1970).

The volume emission rate at 63μ, computed with equation (25), is shown in Fig. 8 for a thermopause temperature of 1000°K. The black body temperature $T_E$ for the lower boundary emission has been taken to be equal to 190°K. This emission is actually responsible for a sign change of $L(z)$: the cooling process becomes a heating mechanism in a region near the mesopause. The heating region can also be seen in Fig. 7, where the slope of the mean net flux becomes negative. Above 100 km, the radiative transfer curve in Fig. 8 is always below the curve corresponding to the optically thin case, in which absorption and induced emission are neglected. If the optically thin approximation is used for computing an atmospheric model, the radiative cooling is overestimated by several orders of magnitude in the region around 100 km. The departure from an optically thin layer is indicated in Fig. 9, where the reduction factor $1 - X$ of expression (25) is shown for two models corresponding to thermopause temperatures of 750°K and 2000°K. The effect of radiative transfer is very important below 150 km and even at greater heights the infrared loss is decreased by 10-20 per cent compared to the case of the optically thin layer.

7. CONCLUSION

The atomic oxygen emission at 63 has been analyzed in the upper atmosphere in relation with the heat conduction equation. As the infrared emission is an important cooling mechanism, the physical processes involved have been described. Absorption and induced emission are important and should be taken into account. A radiative transfer equation has therefore been solved and the results indicate that the optically thin approximation is no longer valid below 150 km. The volume emission rate
Fig. 8.- Volume emission rate for a thermopause temperature of 1000 °K. The heating region is due to the lower boundary emission corresponding to a black body temperature of 190 °K.
Fig. 9.- Reduction factor $1 - X$ showing the departure from an optically thin atmosphere.
obtained by solving the radiative transfer equation is always smaller than in an optically thin layer. Actually, the volume emission rate can change its sign, leading in this way to a heating mechanism around the mesopause.

A detailed solution of the heat conduction equation should take into consideration an infrared loss computed under radiative transfer conditions. If a three dimensional solution is attempted, the problem is even more complex, since the plane parallel geometry must be modified.
REFERENCES